Standard and Extended Form Games

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(Content for slides are adapted from J. Vidal's
Fundamentals of *Multiagent Systems*

Standard and Extended Form Games

- In all multiagent systems we have a set of autonomous agents each performing its own actions using whatever information is has available.

- The other agents are also taking actions, each agent must also take these into account when deciding what to do.

- The agent must decide what to do when their choice of action depends on the others' choices

- A set of mathematical tools has been developed over the years to model and solve these problems. It is known as game theory

Origin of Game Theory

- It was first formally introduced in the book "The Theory of Games and Economic Behavior", (Neumann and Morgenstern, 1944).

- Some game are **noncooperative** because the agents' preferred sets of actions can be in conflict with each other. That is, what is good for one agent might be bad for the others.

Standard and Extended Form Games

- Game theory was first formally introduced in the book "The Theory of Games and Economic Behavior".

- Two most basic forms of games: normal and extended form games.

- These games are known to be noncooperative

- games because the agents' preferred sets of actions can be in conflict with each other.

Games in Normal Form

- Normal form, also known as strategic form.

- In the simplest type of game we have two agents each of which must take one of two possible actions. The agents take their actions at the same time. They will then each receive a utility value, or payoff, based on their joint actions.

- Games such as this one can be represented using a payoff matrix

Games in Normal Form

- Normal form games also assume that the players have common knowledge of the utilities that all players can receive.

- players have common knowledge of

- common knowledge the utilities that all players can receive. That is, everybody knows that everybody

- knows that everybody knows, and so on, the values in the payoff matrix.

		Alice	
		c	d
Bob	a	1,2	4,3
	b	3,2	2,4

Byzantine generals problem

- It is also interesting to note that in message-passing multiagent systems where messages can be lost it is impossible for agents to ever achieve common knowledge about anything.

- The problem is historically described as the *Byzantine generals problem*

- The generals must both attack at the same time in order to defeat the enemy.

- The only method of communication is by sending a messenger who could be captured by the enemy.

strategy

- Normal form game, we define a strategy s to be the set of actions all players take.

- Pure strategy: one where the agents take a specific action.

- Mixed strategy: one where the agents take different actions, each with some fixed probabilities.

Rational

- Game theory assumes that player are rational, which we use as a synonym for selfish. That is, a rational player always acts so as to maximize its utility.

zero-sum

- A special type of game are those in which the values in every box of the matrix add up to zero. These games are known as zero-sum games.

Solution Concepts

- The earliest solution concepts were proposed by Von Neumann.

- He realized that in any given game an agent could always take the action which maximized the worst possible utility it could get. This is known as the maxmin strategy, or minmax

- In a game with two agents,i and j, agent i's maxmin strategy is given by

$$s_i^* = \max_{s_i} \min_{s_j} u_i(s_i, s_j).$$

Minimax theorem

- For the maxmin strategy we have the minimax theorem which states that a strategy that minimizes the maximum loss, a minmax strategy, can always be found for all two-person zero-sum games. Thus, we know it will work at least for this subset of games.

Lack of Stability

- The strategy where both players play their maxmin strategy might not be **stable** in the general case.

- It can happen that if i knows that j will play

- its maxmin strategy then i will prefer a strategy different from its maxmin strategy.

- In the example, the maxmin strategy is (b, d) but if Alice plays d then Bob should play a.

Dominant

- S is the dominant strategy for agent i if the agent is better off doing s regardless of which strategies the others use. Formally, we say that a pure strategy s_i is dominant for agent i if

$$\forall_{s_{-i}} \forall_{r_i \neq s_i} u_i(s_{-i}, s_i) \geq u_i(s_{-i}, r_i),$$

- Where s_{-i} represents the strategies of all agents except i

Iterated dominance

- Dominant can be expanded into the iterated dominance solution in which dominated strategies are eliminated in succession.

Social welfare strategy

- The social welfare strategy is the one that maximizes the sum of everyone's payoffs.

- a social welfare strategy might

- not be stable.

$$s^* = \arg\max_s \sum_i u_i(s).$$

Pareto optimal

- A strategy s is said to be Pareto optimal if there is no other strategy s' such that at least one agent is Pareto optimal better off in s' and no agent is worse off in s' than in s.

- The set of all Pareto strategies for a given problem is formally defined to be the set

$$\{s \mid \neg \exists_{s' \neq s} (\exists_i u_i(s') > u_i(s) \land \neg \exists_{j \in -i} u_j(s) > u_j(s'))\}$$

- In Economics the Pareto solution of often referred to as Pareto efficient or, simply, the efficient solution.

Nash equilibrium

- Strategy s is a Nash equilibrium if for all agents i, s' is $i's$ best strategy given that all the other players will play the strategies in s.

- The set of all Nash equilibrium strategies for a given game is given by

$$\{s \mid \forall_i \forall_{a_i \neq s_i} u_i(s_{-i}, s_i) \geq u_i(s_{-i}, a_i)\}.$$

Nash equilibrium

- There is no general relation between the Nash equilibrium and the Pareto solution.

- The most common problem we face when designing multiagent systems is the existence of multiple equilibria.

Famous Games

- The most famous game of all is the Prisoner's Dilemma

		A	
		Cooperate	Defect
B	Cooperate	3,3	0,5
	Defect	5,0	1,1

Battle of the sexes

- The battle of the sexes is another popular game.

		Alice	
		Ice Hockey	Football
Bob	Ice Hockey	4,7	0,0
	Football	3,3	7,4

Game of chicken

- The game of chicken, is also common.

		Alice	
		Continue	Swerve
Bob	Continue	-1,-1	5,1
	Swerve	1,5	1,1

Repeated Games

- The prisoner's dilemma the dominant strategy was to defect but both players could have received a higher utility by cooperating.

- One way to try to get out of this conundrum, and to better simulate real-world interactions, is to let two players play the same game some number of times. This new game is known as the iterated prisoner's dilemma.

- One way to analyze repeated games that last for a finite number of periods is to backtrack from the end.

Repeated Games

- cooperative equilibrium for the iterated prisoner's dilemma if instead of a fixed known number of interactions there is always a small probability that every interaction will be the last interaction.

Folk theorem

- The folk theorem tells us that in a repeated game any strategy that is not Pareto dominated by another and where every agent gets a utility that is higher than his maxmin utility is a feasible equilibrium strategy.

- In these cases, each player knows that he can get his maxmin utility by playing the appropriate action, regardless of what the others do. Thus, no player will be satisfied with less than his maxmin utility.

Games in Extended Form

- In the early 1980's Robert Axelrod performed some experiments on the iterated prisoner's dilemma.

- He sent out an email asking people to submit fortran programs that played the prisoner's dilemma against each other for 200 rounds.

- The winner was the one that accumulated the most points. Many entries were submitted.

Games in Extended Form

- They included the following strategies:
- ➤ ALL-D: always play defect.
- ➤ RANDOM: pick action randomly.
- ➤ TIT-FOR-TAT: cooperate in the first round, then do whatever the other player did last time.
- ➤ TESTER: defect on the first round. If other player defects then play tit-for-tat. If he cooperated then cooperate for two rounds then defect.
- ➤ JOSS: play tit-for-tat but 10% of the time defect instead of cooperating.
- The tit-for-tat strategy won the tournament.

Games in Extended Form

- In extended form games the players take sequential actions.

- These games are represented using a tree where the branches at each level correspond to a different player's actions and the payoffs to all agents are given at the leafs.

- Extended form games can also represent simultaneous actions by using dotted ellipses

Games in Extended Form
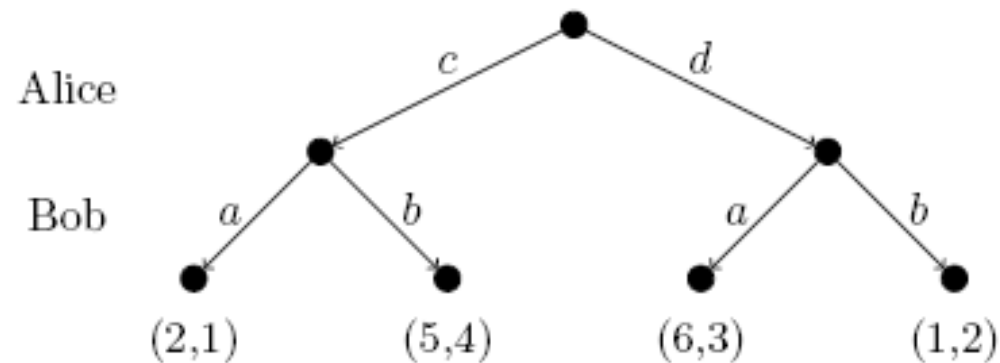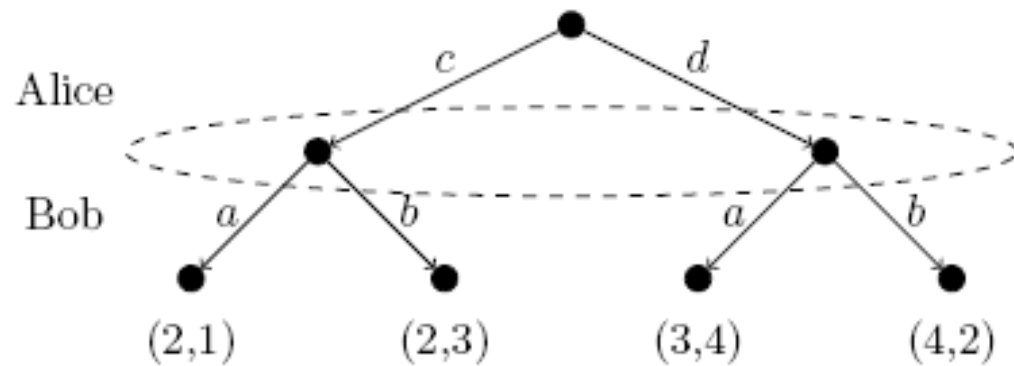
Solution Concepts

- An extended game has a Nash equilibrium strategy s^* if for all agents i it is true that they can't gain any more utility by playing a strategy different from s_i^* given that everyone else is playing s_{-i}^* .

- Extended form games are a useful way to represent more complex multiagent interactions.

Subgame perfect equilibrium

- A stronger solution concept is the subgame perfect equilibrium strategy s^* which is defined as one where for all agents i and all subgames it is true that i can't gain any more utility by playing a strategy different from s_i^* .

Finding a Solution

- There is an extensive literature on centralized algorithms for finding the various equilibrium strategies for a given game.

- The algorithms involve a complete search using heuristics for pruning.

- The Gambit software program is an open source implementation of several of these algorithms.