A Dynamic, Social, Political Network Model

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Let N = {1, 2, …, n} be a finite population of individuals who interact repeatedly in pairs by their participation in a social network. Each agent will be randomly assigned k unary attributes (e.g., actor type or ideology) that will affect their affinity, i.e., mutual attraction, to others. Attributes are used to determine *homophily* among individuals (Jackson, 2008). We will use *Jaccard similarity coefficient* to compute the amount of homophily. For simplicity, we will assume attributes to stay constant over time. In reality, attributes change over time and these changes account for changes in homophily among individuals and in turn they produce links that change over time. As individuals change naturally, their gravitational force for attraction changes accordingly. Homophily might run counter to some social psychology evidence for attraction among opposites (Lieberman, et. al., 2007). However, we will follow the general trend to assume this holds until we consider more complex settings. The canonical social network for N will be an undirected graph $g \subset N * N$. Two nodes are either connected or they are not. It cannot be that one node is related to a second without the second being related to the first.

Initially, we randomly assign each actor to one of two political stances (i.e., opinions) – (a) for violence (i.e., hawkish using Economics terms), or (b) against violence (i.e., pacifist or "dove" using Economics terms). These differ from individual attributes and will be used as messages to be sent to other individuals. In a given time cycle, a *message* is transmitted through a link between nodes A and B only once. Abandoning directional links helps us to avoid complexity of cycles. Although a link lacks direction, in a given time unit, a message is sent once from a sender to a receiver without concern for acknowledgement. For simplicity, we do not consider transmission failures of any type.

Given the set N of agents, connection between each pair of agents is initially guided by a parametric, stochastic value. Links will be bidirectional but lack reflexivity. *Links* are used as the exclusive channels for information exchange. Consider each bilateral interaction as the development of a shared project over time. Given exogenous conditions, interactions will exhibit both long-run benefits from cooperation and short-run gains from opportunistic behavior.

Strength of a link depends on two properties. Frequency of usage is the first property—we assume that the more often a link is used, the stronger it becomes. Therefore, we use frequency as our first component for determining strength of link strength. The second property is homophily. Pairs of connected agents who share multiple attributes will strengthen their link proportional to the force of their homophily. We will use homophily as our second component for determining link strength. Pairs of connected agents who share multiple attributes will strengthen their link by force of homophily, while other links might be weakened or dissolved due to lack of adequate homophily, thus allowing us to develop networks that change shape over time.

Transitivity is an individual node property that can be set to on or off. With it on, messages move forward one unit automatically beyond the recipient but proportional to link strength. This is represented by a coefficient $1/\mathcal{L}$. If a link to a node is an indirect link, strength of that link is multiplied by $1/\mathcal{L}$ where $\mathcal{L}$ is a user determined attenuation parameter.

Transitivity is a global link property that can be set for every node as a toggle switch value; i.e., all links on or all links off. With transitivity set to off, information exchange is dyadic and no indirect links exist. Influence is limited to direct communication. Information received stops going forward beyond the message receiver.

With it set to on, transitive information exchange is hyperdyadic, i.e., going beyond a pair of communicating nodes. For simplicity, we limit transitivity to a single link. Therefore, influence in one time unit is limited to "friend of a friend" and not beyond.

Network architecture, i.e., *topology* or *structure* is the shape of the network. It is user determined. Initial two topologies we will explore are random and scale free networks (Newman, et. al., 2006). Scale free networks were first proposed by Albert Barabasi to reflect the *power law* distribution of connectivity (Barabasi, et. al., 1999).

Beyond initial individual stances, we model changes to these individual states as a function of network interactions that are modeled in terms of simulated, exchanges of political stance among linked pairs.

This will be studied using a period of parametric time period, *T*, that will be used to simulate T distinct exchanges. It is commonly known that influence among individuals dissipates after three degrees of separation (Christakis and Fowler, 2009). Therefore, T will be assigned a number > 3. Within $(n-1)*T$ time units, information will reach the periphery of the network. With a hypothetical X nodes (X < n) leading original opinions, we will need X* (n-1) * T time units to inform all nodes. If X = n, initial time spread phase will be n* (n-1) * T. Transitivity will accelerate dissemination.

In dyadic communication, Contagion will be limited to neighbors (i.e., one degree of separation), thus ensuring that the network is part of the evolution. Individuals will increment opinions they receive from their neighbors. If the opinions are the same as their own, they strengthen their original stance. Otherwise, if the balance of opinions overwhelmingly differs from their own, they will change. A global, default parameter will be used to determine the threshold of opinion stubbornness in the population.

At each node and each cycle of simulation, we gather influence of others by adding up direct and indirect influences of others. For all direct links of a node we add hawk and dove messages. Balance of this value reflects the sum of direct influences. For all indirect links of a node we add hawk and dove messages. Balance of this value multiplied by $1/\mathcal{L}$ reflects the sum of indirect influences. In reality, there will be a natural temporal decay in link strengths as well as influences at each node. However, for simplicity, we will assume that there is no decay.

We will run our simulation by adjusting parametric values and examine the effects of network structures. Keeping all else equal, we test the following five hypotheses:

H1: Scale free networks disseminate opinions more rapidly than random networks.

H2: Without transitivity, opinion diffusion stabilizes (i.e., saturates) in less than n* T units of time.

H3: With transitivity, opinion diffusion is less than twice as fast as without transitivity.

H3: Opinion polarization is evident in scale free networks.

H4: Opinion density mirrors network structure.

H5: Embededededness/centrality of individuals is a predictor of influence (Kearns, 2010).

Externality Extension:

Externality is the amount of an individual's gains and losses due to actions of others in her neighborhood. Let us hypothetically assume that possessing a political opinion has an associated payoff. E.g., if you lived in a tea-party neighborhood you'd be assumed to be patriotic. For a second example, if you lived in a white color neighborhood, you'd be assumed to have a good credit. We could then argue about externality of being near a desirable neighborhood.

H6: Externality of well connected individuals is higher than externality of poorly connected individuals (Redondo, et. al., 2009).

Network Game extension:

Let payoff to agent i be the ratio of consistency of opinions between agent i's and its neighbors' opinions with a range [-1, +1]. Value of 1 is for full consensus and unanimity of opinions. The value of -1 is for full heterogeneity of opinions. Each agent would wish to guess opinions of its neighbors before forming an opinion itself. Agents have complete information on the social network. Fearing to be in the minority and earning low payoffs, no one would want to be the first to announce an opinion. Clearly, guessing majority opinion is impossible. For this extension, links are used for frequent chatter among individuals to arrive at decisions. We relax our earlier assumption that one message is allowed per time unit per link. Equilibrium is when a group of friends that form a connected component simultaneously announce the same opinion and their payoff is equals to 1. It follows that since there are no links between connected components, we would have uniform, disparate opinion groups.

Power/Influence extension:

From exchange theory, we understand that individuals with similar topological, network location property; specifically embeddedness (i.e., centrality) have dyadic power/influence on one another (Easley and Kleinberg, 2010).

Balance Theory Extension:

Fritz Heider's social psychology Balance theory (1946) suggested that links among three nodes are balanced if either (a) all three links represent pairwise positive leanings or (b) the nodes can be divided into two groups, X and Y, such that every pair of nodes in X have a positive relationship, likewise every pair of nodes in Y like each other, and everyone in X is the enemy of everyone in Y.

We extend or model with link strength to range between -1.0 to +1.0 with positive numbers representing homophily and negative numbers to be anti-homophily. This leads us to form the following hypothesis:

H6: Using [-1, +1] links, individuals will gather into two antagonistic bi-polor opinion groups of hawks and doves.

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