A Perfect information Game in Extensive Form GT

$(N, A, H, Z, \chi, \rho, \sigma, u)$

players, actions, nonterminal nodes, terminal nodes

$\chi : H \to 2^A$ $H \cap Z = \phi$

Possible actions at each choice node

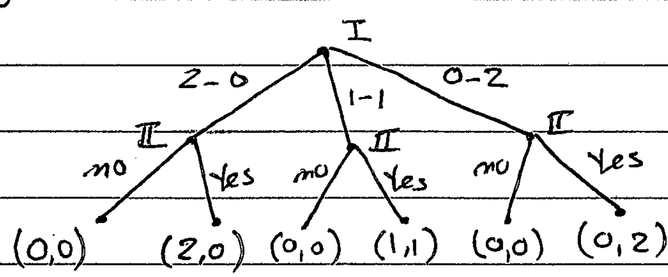
$\rho : H \to N$ assigns players to nonterminal nodes.

$\sigma : H \times A \to H \cup Z$ successor function

$u = (u_1, \ldots, u_n)$ where $u_i : Z \to \mathbb{R}$ assigns utility for player i at a terminal node

E.g. 1

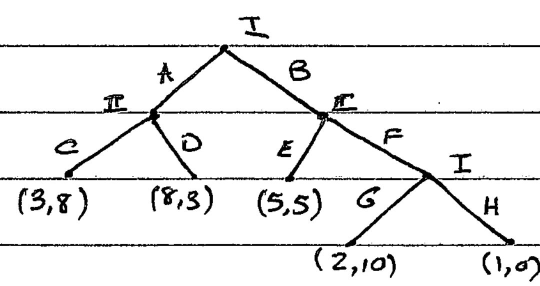
The Sharing Game. I chooses to split 2 gifts. II Chooses to accept or reject



$S_I = \{2\text{-}0, 1\text{-}1, 0\text{-}2\}$

$S_{II} = \{(Yes, Yes, Yes), (Yes, Yes, no), (Yes, no, Yes), (Yes, no, no), (no, Yes, Yes), (no, Yes, no), (no, no, no)\}$

e.g. 2



$S_I = \{(A, G), (A, H), (B, G), (B, H)$

$S_{II} = \{(C, E), (C, F), (D, E), (D, F)\}$

Hexmoor
2010

	(C,E)	(C,F)	(D,E)	(D,F)
(A,G)	3,8	3,8 °	8,3	8,3
(A,H)	3,8	3,8 °	8,3	8,3
(B,G)	5,5	2,10	5,5	2,10
(B,H)	5,5 °	1,0	5,5	1,0

° Nash Equilibrium

Extensive Form ⟶ Normal Form always exists.

Redundancies are present, e.g. (3,8) entries

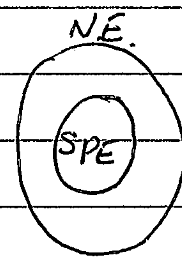
Normal Form ⟶ Extensive Form does not always exist.

Theorem: Every finite perfect information game in extensive form has a pure strategy Nash equilibrium.

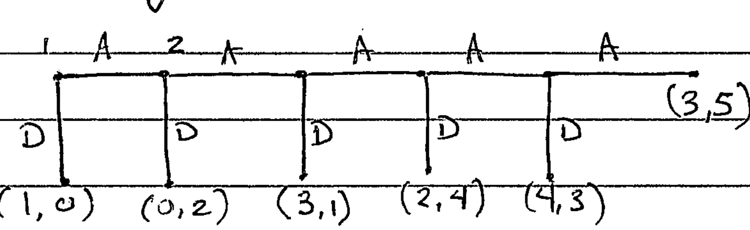
In games of <u>perfect information</u>, you know precisely where in the tree you are located.

A <u>subgame</u> of a game G rooted at h is the restriction of G to the descendants of h.

The subgame perfect equilibria (SPE) of a game G are all strategy profile s such that for any subgame G' of G, the restriction of s to G' is N.E. of G'.



The Centipede game



A – Across

D – Down

The only SPE is for each player to always choose D.

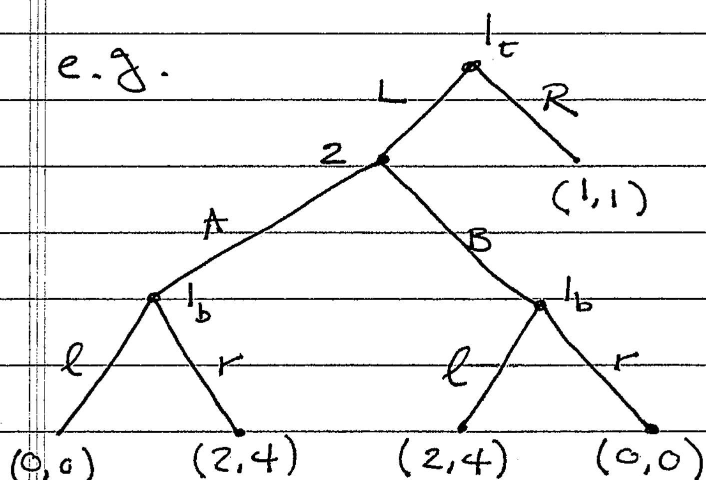
Imperfect Information game

GT

$(\underbrace{N, A, H, Z, \chi, \rho, \sigma, u}_{\text{Perfect-Info game}}, I)$

$I = (I_1, \ldots, I_n)$ where $I_i = (I_{i,1}, \ldots, I_{i,k_i})$ is an equivalence relation on $\{h \in H : \rho(h) = i\}$ where $\chi(h) = \chi(h')$ and $\rho(h) = \rho(h')$ whenever $\exists$ a J for which $h \in I_{i,J}$ and $h' \in I_{i,J}$.

e.g.

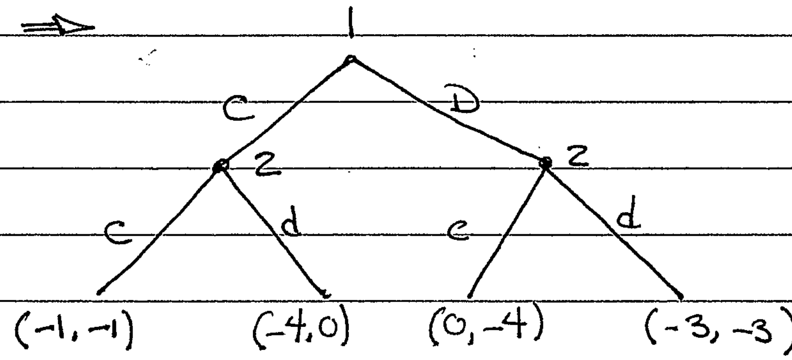


Information sets $= \{1_t, 1_b\}$

At 1_b, 1 has actions l & r regardless of what 2 decides.

Any normal game $\Rightarrow$ imperfect-Information game.

e.g. PD $\Rightarrow$



Behavioral Strategies $\equiv$ A Player's Probabilistic Choice at each node that are independent of other nodes. These are ~~separate~~ distinct from mixed Strategies.

In games of Perfect recall, behavioral & mixed strategies Coincide.

Repeated Games

GT

A *stage game* is played multiple times by the same set of players.

A way to capture repeated games finite times is to show it in an imperfect-information game in extensive form.

Payoffs accumulate over time.

A *stationary strategy* is to reproduce the same strategy profiles repeatedly. This would be memoryless.

In an infinite version, payoffs could be averaged in the limit.

$$\lim_{K \to \infty} \frac{\sum_{J=1}^{K} r_i^{(J)}}{K}$$

$r_i^{(J)}$ is the payoff at round J of the repeated sequence.

With future discount factor of $0 \leq \beta \leq 1$, rewards are

$$\sum_{J=1}^{K} \beta^J r_i^{(J)}$$