$m \boxed{}^{\,n}$ Payoff Matrix GT

$\underset{1 \le J \le n}{\text{Min}}\, a_{iJ}$ Securing level for each row of matrix

$v^- = \underset{1 \le i \le m}{\text{Max}}\ \underset{1 \le J \le n}{\text{Min}}\, a_{iJ} = \underset{1 \le i \le n}{a_{hJ}}$ row h

$\underset{1 \le i \le m}{\text{Max}}\, a_{ij}$ securing level for each column of Payoff matrix

$v^+ = \underset{1 \le J \le n}{\text{Min}}\ \underset{1 \le J \le m}{\text{Max}}\, a_{iJ} = \underset{1 \le i \le m}{\text{Max}}\, a_{ik}$ ~~column k~~

e.g. 1

$$\begin{array}{ccccc|c}
10 & 5 & 5 & 20 & 3 & 3 \\
10 & 15 & 10 & 17 & 25 & 10 \\
7 & 12 & 8 & 9 & 8 & 7 \\
5 & 13 & 9 & 10 & 5 & 5 \\
\hline
10 & 15 & 10 & 20 & 25 &
\end{array}$$

v^- $h=2$

a_{21} a_{23} are equilibria

$v^- = v^+$

v^+ $k = 1$ or 3

e.g. 2

$$\begin{array}{cc|c}
1 & 3 & 1 \\
4 & 2 & 2 \\
\hline
4 & 3 &
\end{array}$$

v^- $h=2$

v^+ $k=2$

$v^- \ne v^+$

Theorem $v^- \le v^+$.

Corollary if $v^- = v^+$, a_{hk} is optimum Pt. $\equiv$ Saddle Point

$\equiv$ Value of game = expected value of game!

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GT

Mixed Strategies

$X = \{x_1, \ldots, x_m\}$ = Probabilities of I Picking Strategies $S_1 \cdots S_m$
$Y = \{y_1, \ldots, y_n\}$ = " " II " " $S_1 \cdots S_n$

Expected Payoff for a game with Payoff matrix $A = \{a_{ij}\}$ is

$$EU = XAY^t = \sum_{1 \le i \le m} \sum_{1 \le j \le n} x_i a_{ij} y_j$$

E.g. $A = \begin{bmatrix} 1 & 3 \\ 4 & 0 \end{bmatrix}$ $v^- = 1$

If $X = \{\frac{1}{2}, \frac{1}{2}\}$ $EU = [\frac{1}{2} \; \frac{1}{2}] \begin{bmatrix} 1 & 3 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

$$= \frac{5}{2} y_1 + \frac{3}{2} y_2$$

$$\geq \frac{3}{2} y_1 + \frac{3}{2} y_2$$

$$= \frac{3}{2}(y_1 + y_2)$$

$$= \frac{3}{2}$$

Security level for I using X_1 is $\quad \text{Min } X_1 A Y^t$ Worst EU for I

" " " II " Y_1 " $\quad \text{Max } X A Y_1^t$ Worst EU for II

notation: for matrix A, $A_{(i)}$ = ith row of A
$A^{(j)}$ = jth column of A.

$XA^{(j)}$ = EU if Player I uses X and Player II uses strategy t_j.

$A_{(i)} Y^t$ = EU if Player I uses strategy S_i and II uses strategy Y.

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theorem: For a fixed strategy X_1 for player 1, GT

$$\text{Min } X_1 A Y^t = \underset{1 \leq J \leq n}{\text{Min}} X_1 A^{(J)}$$

For a fixed strategy Y_1 for player 2,

$$\text{Max } X A Y_1^t = \underset{1 \leq i \leq m}{\text{Max}} A_{(i)} Y_1^t$$

The Minimax Theorem: For every two person, zero sum game, there always exists a mixed strategy for each player such that the expected payoff of one player is the same as the expected cost for the other. $v^- = v^+$ is the best payoff each can expect to receive from the game.

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Eliminating dominated Strategies

	L	C	R
U	0,2	3,1	2,3
M	1,4	2,1	4,1
D	2,1	4,4	3,2

(1) →

	L	C	R
M	1,4	2,1	4,1
D	2,1	4,4	3,2

(2) ↓

	L	C
M	1,4	2,1
D	2,1	4,4

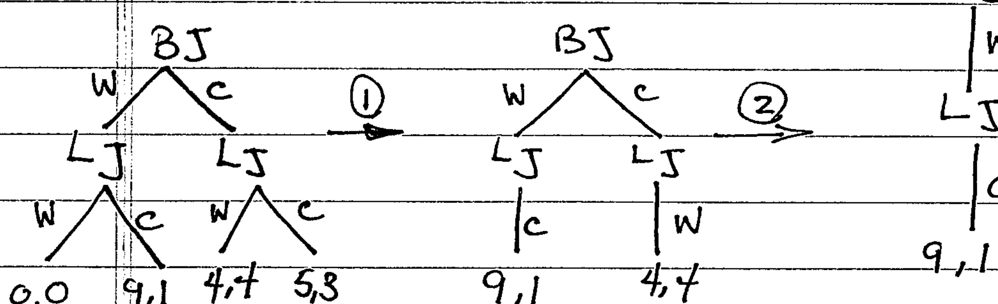
(3) ↓

	L	C
D	2,1	4,4

(4) ↓

D 4,4

Backward induction



W – wait   C – Climb   BJ – Big John   LJ – Little John

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