GT

Game Value

$$\begin{array}{l|cccc|c|l}
 & & \text{II} & & & & \\
\text{Consider} & a_{11} & a_{12} & \cdots & a_{1m} & \text{row min } a_{1J} & v^- = \text{largest min} \\
 & a_{21} & a_{22} & \cdots & a_{2m} & a_{2J} & \\
\text{I} & a_{31} & & & & \vdots & \\
 & \vdots & & & & & \\
 & a_{n1} & a_{n2} & \cdots & a_{nm} & a_{nJ} & \\
\hline
\text{Column max} & a_{i1} & a_{i2} & & a_{im} & & \\
\end{array}$$

$v^+ =$ smallest max

$v^- =$ lower bound of the game. In the worst case, Player I is gurranteed to get v^-. I.e., Player I's payoff floor

$v+ =$ upper bound of the game. In the worst case, Player II is gurranteed to lose at most $v+$ amount. I.e. player II's loss ceiling.

A game has a value iff $v^- = v^+$

$a_{ij} = v^- = v^+$ is the Saddle point (i.e., optimal place) for pure strategies.

Hexmoor
SIUC 2010

Charlie

GT

Auxiliary Diagrams

Ruth

	$1-q$	q
$1-p$	a	b
p	c	d

Theorem: if one player of the game employs a fixed strategy, then the opponent has an optimal strategy that is Pure.

~~Pure~~ Fixed Strategy

Consider

	1	0
$1-p$	a	b
p	c	d

(a)

	0	1
$1-p$	a	b
p	c	d

(b)

$$EU_a = (1-p) \times 1 \times a + p \times 1 \times c = (c-a)p + a$$

$$EU_b = (1-p) \times 1 \times b + p \times 1 \times d = (d-b)p + b$$

$$E_R(p) = \text{the lesser of } \{EU_a, EU_b\}$$

For Penny matching game

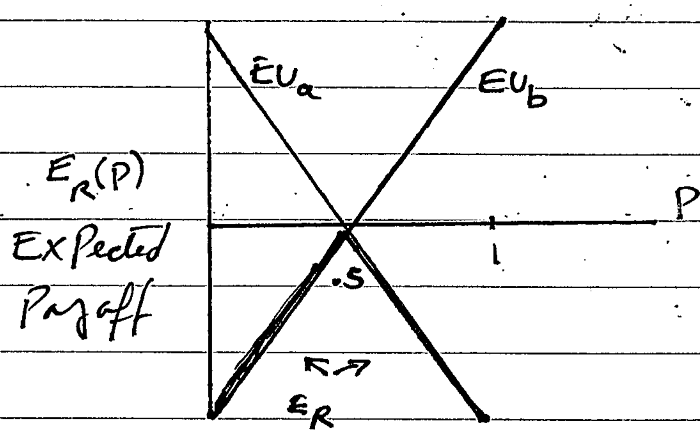
	H	T
H	1	-1
T	-1	1

$a = d = 1$

$b = c = -1$

$EU_a = 1 - 2p$

$EU_b = 2p - 1$

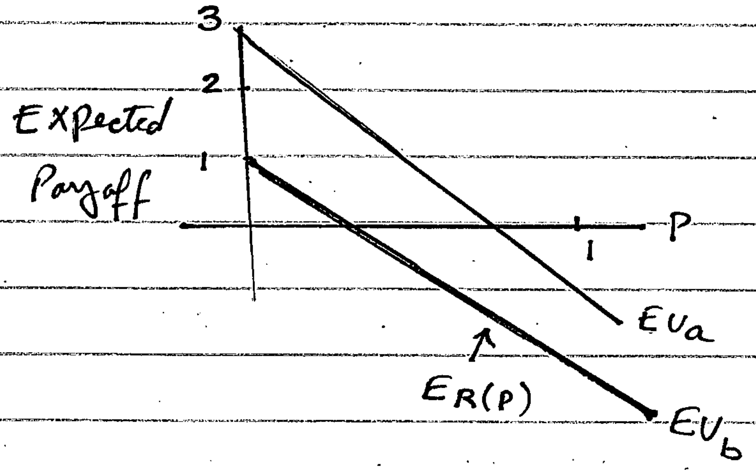


Consider the game

3	1
-1	-2

$EU_a = (-1-3)P + 3 = -4P+3$

$EU_b = (-2-1)P + 1 = -3P+1$

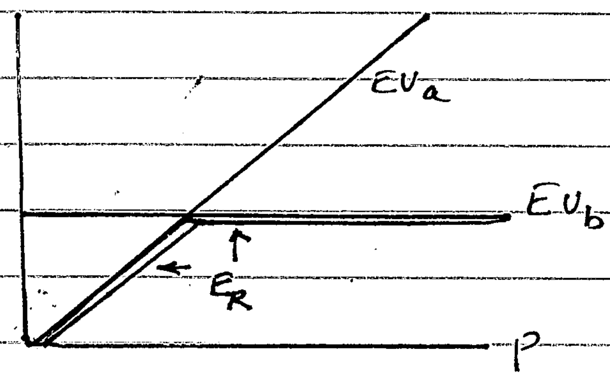


Consider the game

0	1
2	1

$EU_a = 2P$

$EU_b = 1$



Eliminating dominated Strategies

	L	C	R
U	0,2	3,1	2,3
M	1,4	2,1	4,1
D	2,1	4,4	3,2

$\xrightarrow{1}$

	L	C	R
M	1,4	2,1	4,1
D	2,1	4,4	3,2

$\downarrow$ 2

	L	C
M	1,4	2,1
D	2,1	4,4

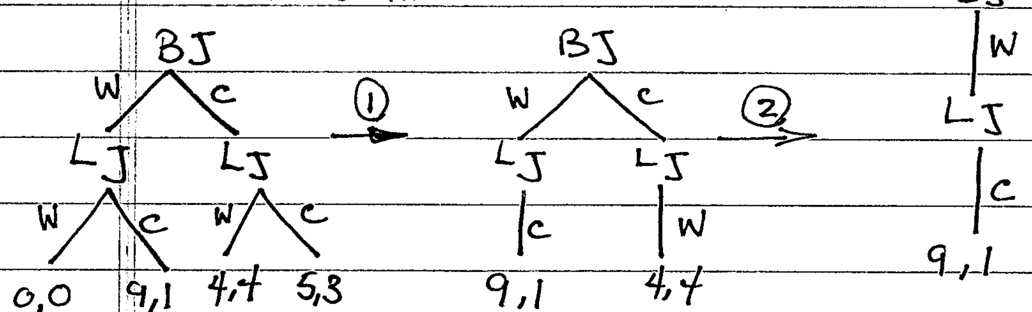
$\downarrow$ 3

	L	C
D	2,1	4,4

$\downarrow$ 4

D 4,4

Backword induction



W – wait C – Climb BJ – Big John LJ – Little John

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