Transmission Control Protocol

GT

TCP Protocol game in Normal Form

when Internet Congestion occurs, backoff from heavy traffic (C)

, Keep on (D)

Your Colleague/Peer

You	C	D
C	-1, -1	-4, 0
D	0, -4	-3, -3

i.e., Prisoner's Dilemma

If ∃ randomness in the environment, games will be bayesian.

If time is involved, extensive games.

Definition: An n person game is a tuple (N, A, u)

N: n players

A: set of action profiles a

$a = (a_1, \ldots, a_n)$

$u = (u_1, \ldots, u_n)$ Payoff / real-valued utility

Prisoner's Dilemma

Outcomes $= A$

generalized TCP:

	C	D
C	a, a	b, c
D	c, b	d, d

Iff $c > a > d > b$

Sometimes $a > \frac{b+c}{2}$

Common-Payoff game: $\forall$ action profile a, $\forall$ agents i, j:

$u_i(a) = u_j(a)$

≡ Pure Coordination game

≡ team game

e.g.,

	s_1	s_2
s_1	1, 1	0, 0
s_2	0, 0	1, 1

Pure Coord ———— Pure Competition

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Zero-Sum game : $\forall$ agents 1 & 2 $u_1(a)+u_2(a)=\phi$ GT
$\equiv$ Pure Competition

Constant-Sum game : $\forall$ agents 1 & 2, $u_1(a)+u_2(a)=C$

e.g., matching Pennies: Pure Competition

	H	T
H	1,-1	-1,1
T	-1,1	1,-1

e.g., Rock, paper, Scissors :

	R	P	S
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0

e.g., Battle of Sexes:

Wife \ Husband	LW	WL
LW	2,1	0,0
WL	0,0	1,2

LW = Lethal weapon
WL = Wonderous Love

Pure Strategy Profile : Select specific action for each player.
Mixed Strategy Profile : randomly choose actions for each player.

Let $\Pi(X)$ = Probability distribution over action set X.

$$S_i = \Pi(A_i)$$

mixed Profile = $S_1, S_2, \ldots, S_n$

$s_i(a)$ = Probability of choosing action a for mixed Strategy S_i

Support of a mixed strategy: A subset of actions that have a positive Probability by the mixed strategy.

Expected utility for a mixed Strategy

$$U_i(S) = \sum_{a \in A} u_i(a) \cdot \prod_{j=1}^{n} S_j(a_j) \quad j \neq i$$

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For a single agent, optimal strategy is one that maximizes its expected payoff. GT

<u>Pareto-domination</u>: A strategy profile s pareto dominates s'
if $\forall i \in N$, $u_i(s) \geq u_i(s')$ and
$\exists j \in N$ for which $u_j(s) > u_j(s')$.

i.e., Some players are made better off without making any other player worse off.

Pareto-optimal: Strategy s is pareto optimal, or strictly pareto efficient if $\nexists s' \in S$ that pareto dominates s.

P. 61
1. $\forall$ game, $\exists$ at least one pareto optimal strategy.
2. Some games will have multiple optima
3. In zero-sum games, all profiles are strictly pareto efficient.
4. In common-payoff games, all pareto optimal strategies have the same payoff.

Define $S_{-i} = (s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n)$: A profile that leaves out agent i's strategy.

<u>Best response</u>: Player i's best response to profile S_{-i} is a mixed strategy s_i^* such that
$$u_i(s_i^*, S_{-i}) \geq u_i(s_i, S_{-i}) \text{ for all } s_i \in S.$$

<u>Nash Equilibrium</u>: A profile $S = (s_1, \ldots, s_n)$ is a Nash equilibrium if, for all agents i, s_i is a best response to S_{-i}.

Nash Equilibrium is <u>stable</u> since no agent would want to change his strategy if he knew what others were following.

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Strict Nash: A Profile $S = (S_1, \ldots, S_n)$ is Strict N
if, $\forall i \; \forall s' \neq s, \; u_i(S_i, S_{-i}) > u_i(S'_i, S_{-i})$

Weak Nash: A Profile S is Weak N
If $\forall i, \forall s' \neq s, \; u_i(S_i, S_{-i}) \geq u_i(S'_i, S_{-1})$ and
S is not a strict N.

e.g. 2 Pure Nash E in Battle of Sexes

		Husband	
		LW	WL
wife	LW	2,1	0,0
	WL	0,0	2,1

Computing mixed strategy Equilibrium

Assume Husband plays LW with Probability P, WL with Prob 1-P.
If wife mixes her strategies, then she must be indifferent between them.

$$U_{wife}(LW) = U_{wife}(WL)$$

$$2 \cdot P + 0 \cdot (1-P) = 0 \cdot P + 1 \cdot (1-P)$$

$$\Rightarrow P = \frac{1}{3}$$

To make the husband indifferent to his actions, wife must choose LW with Prob $\frac{2}{3}$ & WL with Prob $\frac{1}{3}$.

All mixed Strategy E are weak Nash E.

Expected Payoff of both is $\frac{2}{3}$.
Pure Strategy Nash dominates mixed Strategy.

e.g. matching Pennies does not have a Pure Nash E.

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P72 **Theorem**: Every game with a finite number of players GT
(Nash, 1951) and action profiles has at least one Nash Equilibrium.

Definition: The maxmin strategy for player i is $\arg\max_{s_i} \min_{s_{-i}} u_i(s_i, s_{-i})$, and the maxmin value for player i is $\max_{s_i} \min_{s_{-i}} u_i(s_i, s_{-i})$.

P74 **Theorem** (Von Neumann, 1928): In any finite, two player zero-sum game, in any N.E. each player receives a payoff that is equal to both his maxmin value and his minmax value.

Regret: An agent i's regret for playing an action a, if the other agents adopt action profile a_{-i} is defined as

$$\underbrace{\left[\max_{a_i' \in A} u_i(a_i', a_{-i})\right]}_{\text{the best response it could have had}} - \underbrace{u(a_i, a_{-i})}_{\text{actual}}$$

max regret

$$\max_{a_{-i} \in A_{-i}} \left(\underbrace{\left[\max_{a_i' \in A_i} u_i(a_i', a_{-i})\right]}_{\text{best possible response}} - \underbrace{u(a_i, a_{-i})}_{\text{actual}} \right)$$

Minimax regret

$$\arg \min_{a_i \in A_i} \left[\max_{a_i' \in A_i} u_i(a_i', a_{-i}) - u_i(a_i, a_{-i}) \right]$$

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