Two individuals are contesting a resource with marginal gain of V. GT L3

	Hawk	Dove
Hawk	$\frac{1}{2}(V-C)$	V
Dove	$\emptyset$	$V/2$

Hawk v. Hawk: 50% injury, 50% being injured.
Hawk v. Dove: Dove retreats, Hawk gets it
Dove v. Dove: they share resource for $V/2$.

P = frequency of H strategist in the population.
$W(H)$: fitness of H strategists
$W(D)$: " " D "
$E(H,D)$: Payoff to H against a D opponent

$$W(H) = W_0 + P \cdot E(H,H) + (1-P)\, E(H,D)$$
$$W(D) = W_0 + P \cdot E(D,H) + (1-P)\, E(D,D)$$

Individuals reproduce their own kind asexually in numbers proportional to their fitnesses.

P' = frequency of Hawks in the next generation.

$$P' = P \cdot W(H) / \bar{W}$$

$$\bar{W} = P \cdot W(H) + (1-P)\, W(D).$$

Population Dynamics

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GT
L3

If I is a stable strategy, if almost all population adopt I, then the fitness of members is greater than any possible mutant.
P = frequency of mutants adopting strategy $J \neq I$.

$$W(I) = W_0 + (1-P)E(I,I) + P E(I,J)$$
$$W(J) = W_0 + (1-P)E(J,I), P.E(J,J)$$

$$W(I) > W(J), \text{ since } P \ll 1$$

$$\text{either} \begin{cases} E(I,I) > E(J,I) \text{ or} \\ E(I,I) = E(J,I) \text{ and } E(I,J) > E(J,J). \end{cases}$$

Standard Conditions for ESS

E.g., in Hawk/Dove game; D is not ESS since $E(D,D) < E(H,D)$
H is a ESS if $\frac{1}{2}(V-C) > 0$ or $V > C$.
If $V < C$, H or D are not ESS.

If I is a mixed ESS, it would have nonzero probability for pure strategies A, B, C, ..., then

$$E(A,I) = E(B,I) = E(C,I) = \cdots = E(I,I)$$

Bishop & Cannings (1978)

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For H&D game, to find mixed ESS, solve

GT
L3

$$E(H, I) = E(D, I)$$

$$P \cdot E(H,H) + (1-P)\, E(H,D) = P \cdot E(D,H) + (1-P) \cdot E(D,D)$$

$$\frac{1}{2}(V-c)P + V(1-P) = \frac{1}{2}V(1-P)$$

$$\therefore P = \frac{V}{c}$$

if

	I	J
I	a	b
J	c	d

mixed ESS exists if $a < c$ & $d < b$,

$$P = \frac{(d-b)}{(b+c-a-d)}$$

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