A binary relation $\odot$ on set $A \subset A \times A$ A = a choice set.

$(x,y) \in \odot$ or $x \odot_A y$ ← Proposition forming; e.g. taller

e.g. arithmetic "$<$" is a binary relation

" "$+$" is not a binary relation, $x + y$ is not a Proposition

Consistency Conditions

A Preference ordering $\geq_A$ is a binary relation with Properties

1. Complete. $\forall x, y$ $x \geq_A$ or $y \geq_A x$.
2. Transitive $\forall x, y, z$, If $x \geq_A$ and $y \geq z \Rightarrow x \geq z$.
3. Independent of irrelevant alternatives
   $\forall x, y \in B$, $x \underset{B}{\geq} y \underset{\text{iff}}{\Leftrightarrow} x \underset{A}{\geq} y$.

x is weakly Preferred to y ≡ An Individual chooses $x \in A$ such that $\forall y \in A$, $x \geq y$.

Completeness implies reflexivity.

Strong Preference is denoted by $>$.

Weak Preference is " " $\geq$.

$x \simeq y$ = ~~equivalent~~ iff $x \geq y$ & $y \geq x$.

exclusion Condition ≡ If $x > y$ then $\neg(y > x)$.

A Preference relation is Consistent iff it meets the Consistency Conditions

Theorem: A binary relation $\geq$ on A of Payoffs can be represented by a Preference function $u: A \rightarrow \mathbb{R}$ iff $\geq$ is Consistent.

Exponential Discounting

Let x & y be two Consumption streams

$x = x_0, x_1, \ldots$ $y = y_0, y_1, \ldots$

Let $\delta \in (0,1)$ be a Constant capturing a discount factor.

$$U(x_0, x_1, \ldots) = \sum_{k=0}^{\infty} \delta^k u(x_k)$$

Let $\delta = e^{-r}$

r = one period, continuous-compounding interest rate.

$$U(x_0, x_1, \ldots) = \sum_{k=0}^{\infty} e^{-rk} u(x_k)$$

If interest accrues at the end of the period, then $\delta = \frac{1}{(1+r)}$

$$U(x_0, x_1, \ldots) = \sum_{k=0}^{\infty} \frac{u(x_k)}{(1+r)^k}$$

<u>Expected Utility Principle</u>

Let $X = \{x_1, x_2, \ldots, x_n\}$ be a set of prizes (n is a finite number!)

$P(x)$ = Probability that the payoff is $x \in X$.

$$\underset{\text{lottery } l}{E[l]} = \sum_{i=1}^{n} \underset{\text{Prob of payoff } x_i}{P_i} \cdot x_i$$

Prize x_i = Payoff value of x_i.

Using the law of large numbers: as the # of times a lottery is played $\to \infty$, the average payoff converges to expected value of the lottery with probability 1, (Feller, 1950).

E.g. if $X = \{\underset{P}{a}, \underset{1-P}{b}\}$ $E[l_1] = P \cdot a + (1-P) \cdot b$

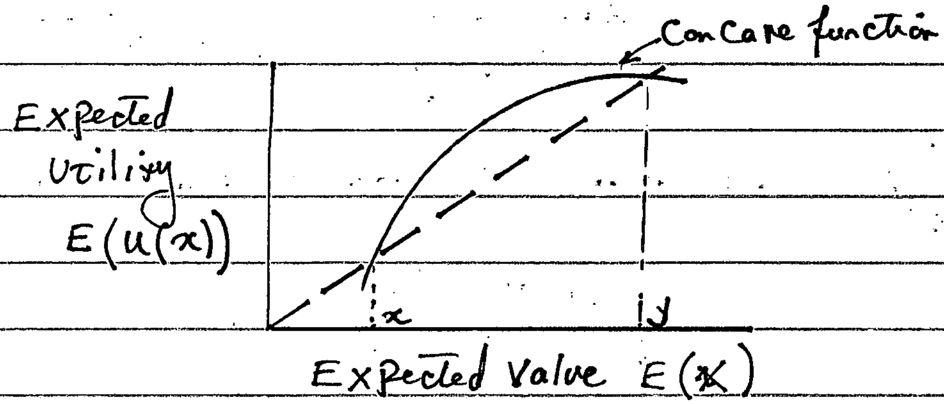
if $X = \{\underset{P}{a}, \underset{q}{b}, \underset{1-P-q}{c}\}$ $E[l_2] = Pa + qb + (1-P-q)c$

if $X = \{a_1, \ldots, a_n\}$ $E[l_3] = P_1 a + P_2 a_2 + \cdots + P_n a_n$

$$= \sum_{i=1}^{n} P_i a_i$$

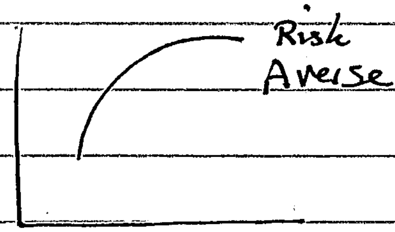
... Bayesian Rationality ...

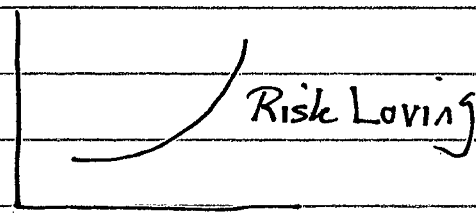
If $X = \mathbb{R}$, Payoffs can be considered to be money.

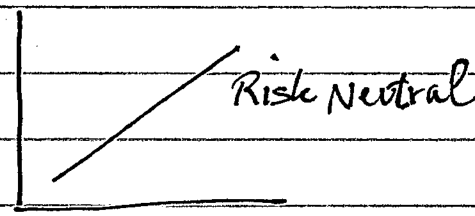


$\forall x, y \in X$,

$$p\,u(x) + (1-p)\,u(y) < u(px + (1-p)\cdot y)$$







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A normal form game

		II		
		L	m	R
	T	(1,1)	(0,2)	(2,1)
I	M	(2,2)	(1,1)	(0,0)
	B	(1,0)	(0,0)	(-1,1)

Von Neumann/Morgenstern utilities for players I & 2.

Payoffs are Common Knowledge.
For I, M > B, For II, ∄ a dominant Strategy.

<u>Stable outcome</u> ≡ Strategy Profiles where no player has <u>incentive</u> to deviate if he knows that the other players play the Prescribed strategies.

E.g.,

	L	R
T	(1,1)°	(0,0)*
B	(0,0)†	(1,1)°

* unstable I ← II ↓

† unstable I ↑ II →

° stable

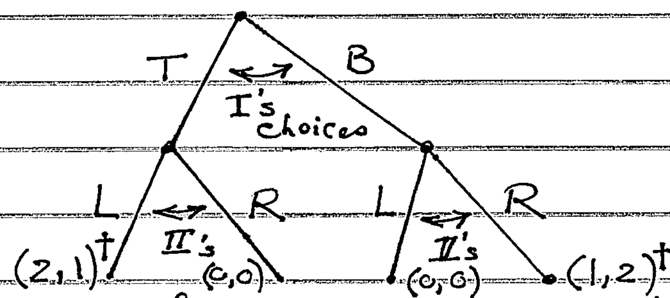
<u>Battle of Sexes:</u>

<u>Normal Form</u>

		II	
		L	R
I	T	(2,1)°	(0,0)
	B	(0,0)	(1,2)°

° Stable outcomes (equilibria)

<u>Extensive form</u>



If this was sequential, i.e. if II chooses after I chose, then II would have chosen † options. Assume I can envision this <u>counterfactual</u> mentally. Then I would select T for his choice. This reasoning is known as <u>backward induction</u>.

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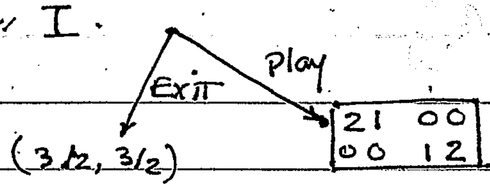
Exiting in the battle of sexes will yield 3/2 for I & 2.
Consider the options for I.



Forward induction: I wants to do better than 3/2, so I playes, hoping for (2,1) equilibrium.

Hawk-Dove game (HD)

	Hawk	Dove
Hawk	$\left(\frac{V-C}{2}, \frac{V-C}{2}\right)$	$(V, 0)$
Dove	$(0, V)$	$\left(\frac{V}{2}, \frac{V}{2}\right)$

i.e., a biological game

V = Value of a resource to be enjoyed by a player

Sharing the resource yields $\frac{V}{2}$.

Hawk = tough stance

C = Cost of a conflict with opponent.

If $V > C$, HD becomes the Prisoner's Dilemma, They go for conflict. Cost is relatively low.

If $V < C$, HD becomes the game of "chicken" (movie Rebel without a Cause)

— more generally, this is "Wars of attrition" games. Here fighting is too expensive.

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