Mechanism Design (MD) GT

Strategic social choice, Implementation theory, Inverse game theory

MD Produces games that have intended equilibria for Players. Designer determines player choices & consequences. No coercion is allowed. Designer's outcome depends on player's choices. (Dutta, 1999)

E.g. 1 – the Commons Problem --

A. Government auctions off the rights to the common resource to highest bidder each year. i.e., Restricted Privatization.

B. the government auctions off the rights to extract from the common once & for all... i.e., Privatization

E.g. 2 Selling the davinci Diaries

Assume Sotherby's is selling; Bill Gates is the most Prominent buyer. Gates' worth; i.e., Private value? Either an aficionado or just a fan.

Sotherby options:

1. Post a high aficionado Price & risk losing a sale.
2. Post Two Prices: At high Price, gurrantee sale
   At a low Price, item might be withdrawn from sale.

optimal mechanism is when expected sale Price is maximum.

E.g. 3 Public auction -- Sotherby's options:

1. English auction – 1st Price open cry
2. 2nd Price (Vickrey)

Analysis of E.g. 2 with Gates only.

Let θ be utility if Gates is an aficionado

Let μ " fan $(\theta > \mu > 0)$

P = Price Paid

$\theta - P$ = Surplus as an aficionado

$\theta - \mu$ = " a fan

GT

ρ = Sotheby's guess that Gates is an aficionado

Sotheby's expected value = $\rho\theta + (1-\rho)\mu$

Special numbers Case: $\theta = \$40M$, $\mu = \$10M$, $\rho = \frac{1}{2}$

S's expected revenue = $\rho\theta + (1-\rho)\mu = \$25M$.

If Sotheby does not know Gates interest level, options:

1. Ask the buyer & charge accordingly; $P(\theta)$ = Price if aficionado
   $P(\mu)$ = " " fan

   If $P(\theta) > P(\mu)$, buyer will not report to be an aficionado
   Then Prices will be set at $P(\mu)$.

   If $P(\theta) = P(\mu)$, buyer will be truthful.

2. Set a flat rate of P.

3. Guarantee sale at a high Price, say $\frac{\theta}{2}$

   At low Price, there could be 50% chance of withdrawl.

S's expected value = $\rho\frac{\theta}{2} + (1-\rho)\frac{\mu}{2}$; for SN, EV = \$12.5M

4. Guaranteed sale at a high price of P

   Prob of withdrawl is Q to sell at a low Price of q. Q = Prob of keeping $\frac{Q}{1-Q}$

aficionado would like it if $\theta - P \geq Q\cdot(\theta - q)$

i.e. $\theta \geq \frac{P - Qq}{1-Q}$

fan would like it if $Q(\mu - P) \geq \mu - P$

i.e. $\mu \leq \frac{P - Qq}{1-Q}$

Altogether $\theta \geq \frac{P-Qq}{1-Q} \geq \mu$ (22.1 Dutta)

Incentive Compatibility Constraint.

To prevent Coercion $\theta \geq P$, $\mu \geq P$

Expected value = $\rho P + (1-\rho)Qq$

Revelation Principle

A mechanism is a game (or a set of rules) that specifies the strategies that the player can choose from and the outcome for each choice. (e.g., a teacher / Parent)

S = a Strategy ; e.g. accept high Priced Sure offer or low Price Uncertain offer

O = an outcome ; e.g., buyer Pays $\frac{\theta}{2}$ & gets the item.

$\pi(S, \tau, \theta)$ = Payoff a player Type θ Playing S for outcome τ

This is outside designer control.

Let (S^*, τ^*) for θ Player, (S', τ') for μ player.

This assignment is incentive Compatible if

$$\pi(S^*, \tau^*, \theta) \geq \pi(S, \tau, \theta) \quad \forall S, \tau$$

$$\pi(S', \tau', \mu) \geq \pi(S, \tau, \mu) \quad \forall S, \tau$$

i.e. θ Prefers S^*, τ^* & μ Prefers S', τ'.

Each player has an outside option, non MD option, π_0.

MD is free of Coercion when

$$\left.\begin{aligned} \pi(S^*, \tau^*, \theta) &\geq \pi_0 \\ \pi(S', \tau', \mu) &\geq \pi_0 \end{aligned}\right\} \text{ Individual rationality Constraints.}$$

Designer of MD is also known as the Principal.

Direct-revelation Mechanism: A mechanism where the strategy set of a player is Simply to report of its type. players see to their interest to tell the truth.

Proposition: For any mechanism & an incentive Compatible, individually rational assignment, there is a direct-revelation mechanism in which truth telling is incentive Compatible. the Principal can restrict attention to direct-revelation mechanisms & truth telling assignments within those mechanisms.