

# Social Choice Theory

choices

GT

Methods for blending individuals to group choices.

e.g., The Congress, The Committee, The Media, The Nation...

often nonstrategic, i.e., free from manipulation (unrealistic)

Plurality voting: 1. Each person votes for preferred choice, 2. Tally votes, 3. Select highest ranked choice

Condorcet Condition:

$\exists$  choice  $x$  such that  $\forall y \neq x$ , at least half prefer  $x$  to  $y$ .

Social choice function  $C: L^a \rightarrow O$   
↑ a set of preferences      ↑ outcomes  
Total order

A choice fn satisfies Condorcet if it always picks a Condorcet choice provided it exists.

Smith Set  $\equiv$  Smallest set  $S \subseteq O$  such that

$$\forall o' \notin S, \#(o > o') \geq \#(o' > o)$$

It always exists. A Condorcet winner is a singleton Smith Set. Otherwise, Smith Set = "stalemate" choices.

Cumulative voting: Each voter has  $k$  votes, can arbitrarily. The candidate with most votes is selected.

Approval voting: Each voter casts a single vote for preferred choice. The candidate with most votes is selected.

Plurality with elimination (i.e. instant run off): Each voter casts one vote for most preferred. The candidate with fewest votes is eliminated. The wasted vote is recast. Repeat until one candidate remains.

Borda: Each voter fully orders Candidates & assigns GT  
Points. For  $n$  Candidates, TOP most gets  $n-1$ , then  $n-2, \dots$   
0 to lowest. Winner is one with most points.

Pairwise Elimination: A list of Pairs of Candidates exist.

Each voter orders a Pair of Candidates.

Candidate with minority preference is eliminated

Repeat until one Candidate remain.

Non-dictatorship:  $W$  does not have a dictator if

$$\nexists i \forall o_1, o_2 (o_1 \succ_i o_2) \rightarrow (o_1 \succ_w o_2)$$

A dictator is an agent whose preferences always determines social ordering.

Theorem (Arrow, 1951): If  $|O| \geq 3$ , any social welfare for  $W$  that is Pareto efficient & irrelevant of indep alt. is dictatorial.