Bayesian game (N, G, P, I) ① GT

N – set of agents

G – set of games with N agents with the same strategy space

$P \in \Pi(G)$ is a <u>Common Prior</u> over games. $\Pi(G)$ is the set of all probability distributions over G,

$I = (I_1, \dots, I_n)$ is a tuple of partitions of G, one for each agent.

e.g.

$I_{2,1}$ / $I_{2,2}$

$I_{1,1}$: Matching Pennies / PD

2,0	0,2
0,2	2,0

2,2	0,3
3,0	1,1

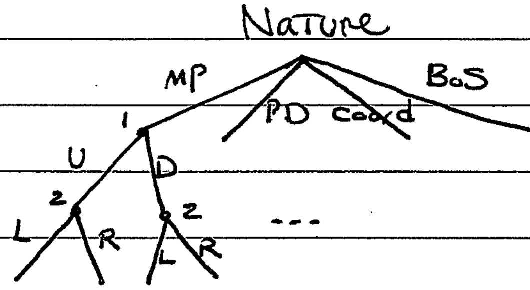
$I_{1,2}$: Coordination / B of Sexes

2,2	0,0
0,0	1,1

2,1	0,0
0,0	1,2

A Bayesian game

Common Prior Conceptualization



② (N, A, Θ, P, u)

$\Theta = \Theta_1 \times \dots \times \Theta_n$, agent types – A type encapsulates all information possessed by the agent that is not common knowledge (includes beliefs about others' beliefs on payoffs) nested beliefs...

$P: \Theta \to [0,1]$ is a common prior over types.

$u = (u_1, \dots, u_n)$, utility functions.

ex post expected utility: based on all agent's actual types

$$EU_i(s,\theta) = \sum_{a \in A} \left(\prod_{j \in N} s_i(a_j \mid \theta_j) \right) \cdot u_i(a,\theta)$$

ex interim expected utility: an agent knows his own type, but not the types of others.

$$EU_i(s,\theta_i) = \sum_{\theta_{-i} \in \Theta_{-i}} P(\theta_{-i} \mid \theta_i) \sum_{a \in A} \left(\prod_{j \in N} s_j(a_j \mid \theta_j) \right) \cdot u(a, \theta_{-i}, \theta_i)$$

$$= \sum_{\theta_{-i} \in \Theta_{-i}} P(\theta_{-i} \mid \theta_i) \cdot EU_i(s,(\theta_i, \theta_{-i})).$$

Ex ante expected utility: agent does not know anybody's type

$$EU_i(s) = \sum_{\theta \in \Theta} P(\theta) \cdot \sum_{a \in A} \left(\prod_{j \in N} s_j(a_j \mid \theta_j) \right) \cdot u_i(a,\theta)$$

$$= \sum_{\theta \in \Theta} P(\theta) \cdot EU_i(s,\theta)$$

E.g. Terrorist Hunt (Meirowitz, 2009)

		CIA	
		Kingpin	Operative
FBI	Kingpin	2, 2	0, 1
	Operative	1, 0	1, 1

CIA's alternate Perception for FBI ①

	K	O
K	2, 2	0, 1
O	0, 0	0, 1

CIA's alternate Perception for FBI ②

	K	O
K	1, 2	0, 1
O	2, 0	2, 1

A Probabilistic Revision

	K	O
K	$10\times\frac{1}{5}+0\times\frac{4}{5},\ 10\times\frac{1}{5}+0\times\frac{4}{5}$	$0,\ 6\times\frac{1}{6}+0\times\frac{5}{6}$
O	$6\times\frac{1}{6}+0\times\frac{5}{6},\ 0$	$6\times\frac{1}{6}+0\times\frac{5}{6},\ 6\times\frac{1}{6}+0\times\frac{5}{6}$