GT L1

Background -- A mini course in Decision theory (DT)

Let X be a random variable to denote an uncertain condition probabilistically.
Let x_j be a possible value for X, $0 \leq x_j \leq 1$.

$$\sum P_r(x_j) = 1$$

Let S be a set of outcomes, i.e. states of the world.
U_i = a set of values (i.e. utilities) agent i places on outcomes.
Von Neumann & Morgenstern (1944) showed that utilities capture an agent's ordering of outcomes, i.e., preferences.
D_i = a set of decisions agent i makes.
Expected Utility = a weighted average of utilities over outcomes.

$$EU = \sum_{S_j \in S} \underbrace{P_r(S_j / D_i)}_{\substack{\text{Conditional} \\ \text{Probability if } D_i \text{ is selected}}} \cdot \underbrace{U(S_j)}_{\text{relative utility of outcome}_j}$$

$$\text{optimal decision} = D^* = \arg\max_{S_j \in S} \sum P_r(S_j / D_i) \cdot U(S_j)$$

DT is used by <u>Self-interested</u> persons to make optimal decisions in uncertain environments. DT is a "game" against nature. Nature behaves randomly. If opponents are other, independent, self interested persons, a multi person DT (MDT) can be considered. This is the premise for GT. Von Neumann & Morgenstern (1944) also gave us an economic foundation for GT.

Self-interested persons are not "selfish" or ill-tempered. They have an ego-centric perspective. Emotions are not considered. GT people are not "heartless". They assume rationality to mean self regard. Herbert Simon -

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A Game = N players, a set of strategies for each player,
{o | o is outcome of each "play" of the game},
o → {P | P is a payoff for each player}

A strategy = each player's plan for decisions known at start of game

A Zero-Sum game = $\sum_{i=\text{player}} \text{Payoff}_i = \emptyset$.

A Constant-Sum game = $\sum_{i=\text{player}} \text{Payoff}_i = C$

A game matrix = Payoff matrix

		Player II		
		S_1	S_2 ---	S_m
player I	S_1			
	⋮		$\boxed{a_{ij}}$ ← Payoff, typically for player I.	
	S_n			

A few examples:

1. Baseball pitch

		Pitcher		
		Fast	Curve	Slide
Batter	F	.30	.25	.20
	C	.26	.33	.28
	S	.28	.30	.33

30% chance of getting a hit for a batter expecting a fastball & a pitcher that throws one. i.e., Batter's payoff.

In a zero-sum game, Pitcher's Payoff = 1 − .3 = 0.7

2. Baby 2x2 Nim

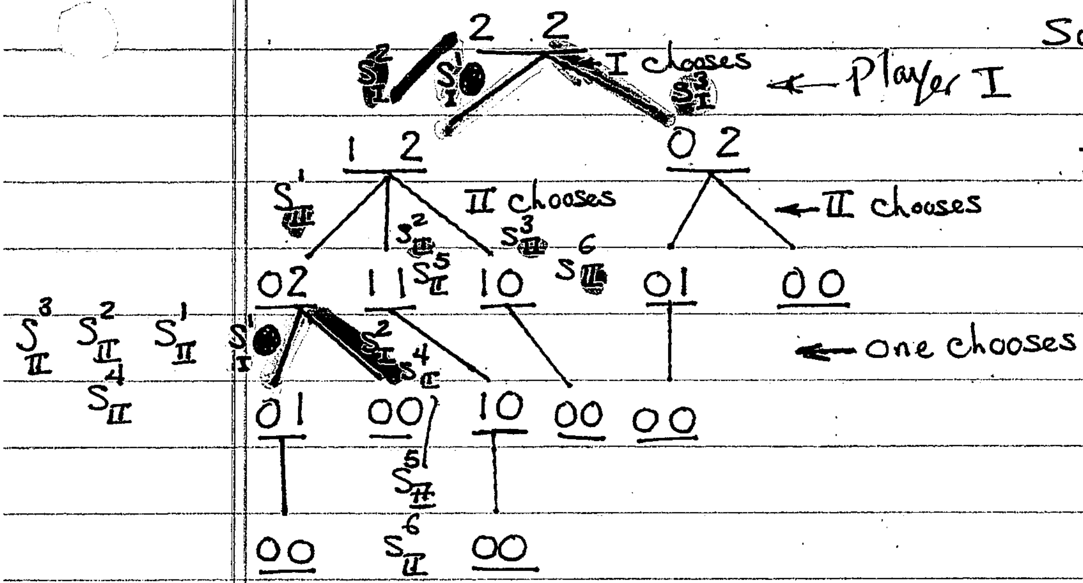
4 pennies in 2 piles of 2 pennies each.

I chooses a pile, remove 1 or 2 pennies.

II choose a pile with at least one penny, remove 1 or 2 pennies.

end when both piles are gone

loser = player removing last penny.



Successive move = extensive form

I has 3 Strategies S_I^1 S_I^2 S_I^3

II : S_{II}^1 S_{II}^2 S_{II}^3 S_{II}^4 S_{II}^5 S_{II}^6

S_{II}^5 is dominated; i.e. loses in all cases.

		II					
		1	2	3	4	5	6
I	1	1	1	−1	1	1	−1
	2	−1	1	−1	−1	1	−1
	3	−1	−1	−1	1	1	1

∃ a wining strategy for player II.

Saddle Points i.e, Optimal Strategies

S_{II}^5 is Dominated

game value = −1

Similar to Tic-Toe-Toe ; not very interesting!

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