Robotic Motion Planning: Bug Algorithms

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What's Special About Bugs

- Many planning algorithms assume global knowledge
- Bug algorithms assume only *local* knowledge of the environment and a global goal
- Bug behaviors are simple:
  - 1) Follow a wall (right or left)
  - 2) Move in a straight line toward goal
- Bug 1 and Bug 2 assume essentially tactile sensing
- Tangent Bug deals with finite distance sensing

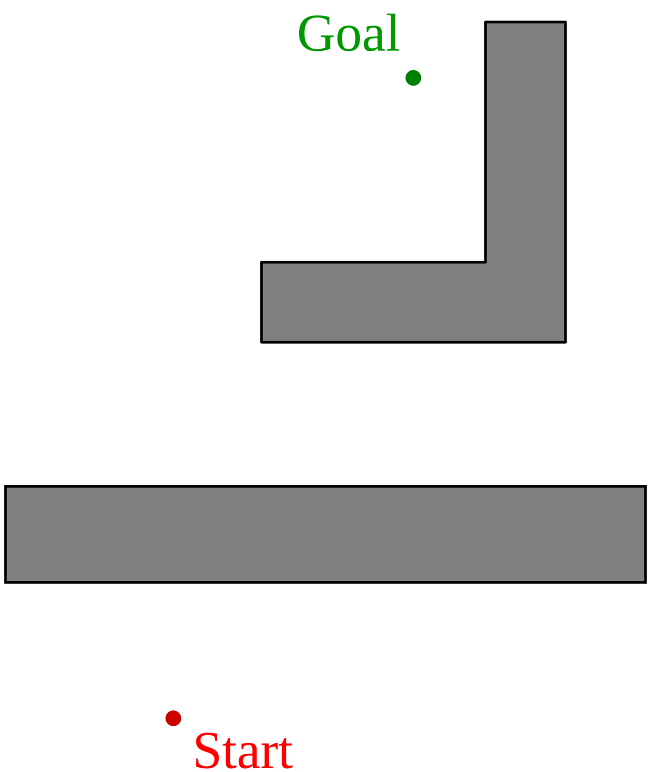
A Few General Concepts

- Workspace W
  - $\Re(2)$ or $\Re(3)$ depending on the robot
  - could be infinite (open) or bounded (closed/compact)
- Obstacle WO_i
- Free workspace $W_{free} = W \setminus \cup_i WO_i$

The ***Bug*** Algorithms

provable results...

Insect-inspired



- **known direction to goal**
  - **robot can measure distance d(x,y) between pts x and y**
- **otherwise local sensing**

  walls/obstacles & encoders
- ***reasonable* world**

  1) finitely many obstacles in any finite area

  2) a line will intersect an obstacle finitely many times

  3) Workspace is bounded

  $W \subset B_r(x),\ r < \infty$

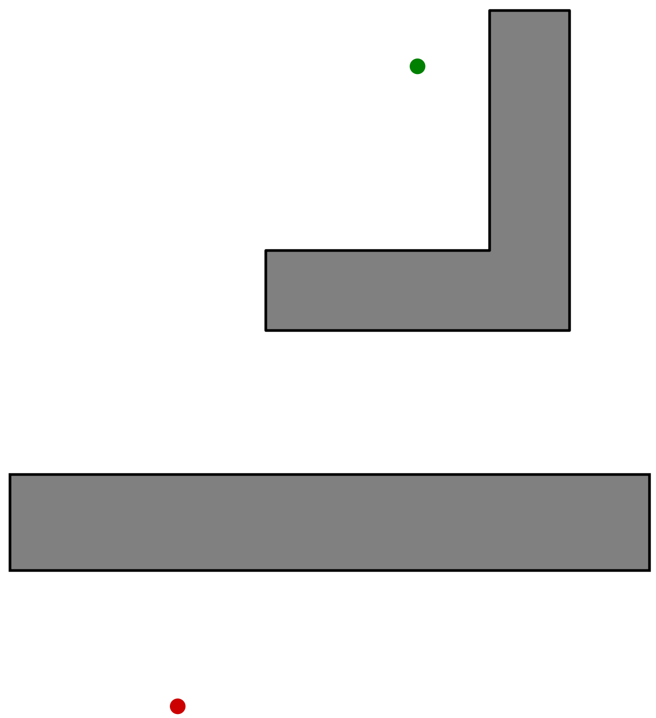
  $B_r(x) = \{\ y \in \Re(2) \mid d(x,y) < r\ \}$

Buginner Strategy

"Bug 0" algorithm

- **known direction to goal**
- **otherwise local sensing**

  walls/obstacles & encoders



Some notation:

q_{start} and q_{goal}

"hit point" q^H_i
"leave point q^L_i

A *path* is a sequence of hit/leave pairs bounded by q_{start} and q_{goal}

how ?

Buginner Strategy

"Bug 0" algorithm

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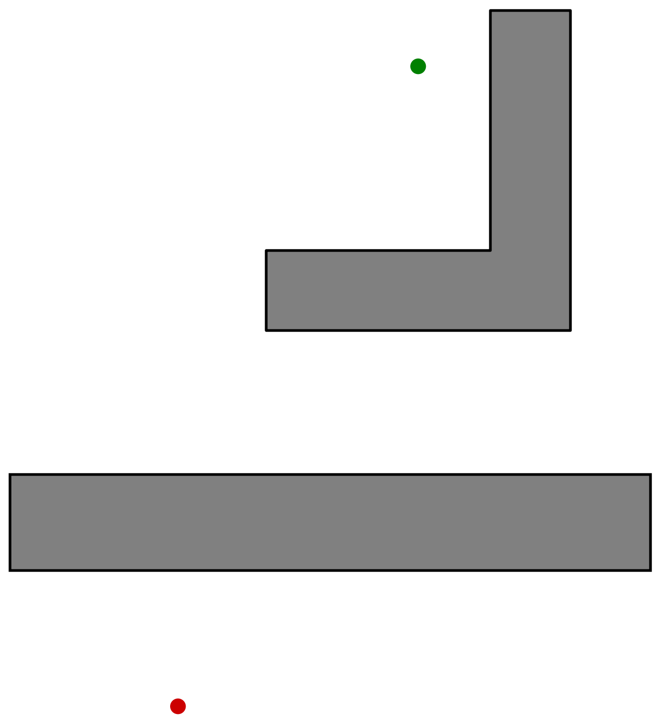
  walls/obstacles & encoders



1) head toward goal

2) follow obstacles until you can head toward the goal again

3) continue

path ?

Buginner Strategy

"Bug 0" algorithm

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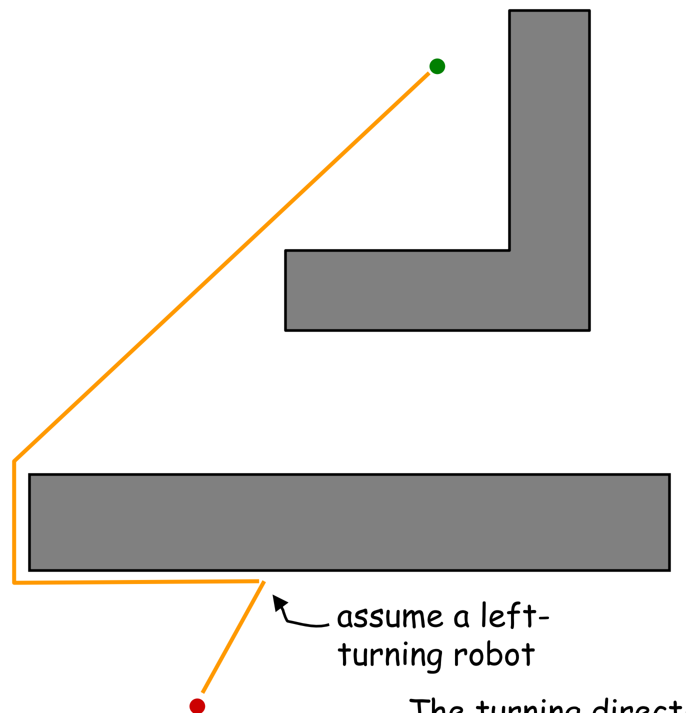


The turning direction might be decided beforehand...

OK ?

Bug Zapper

What map will foil Bug 0 ?

"Bug 0" algorithm

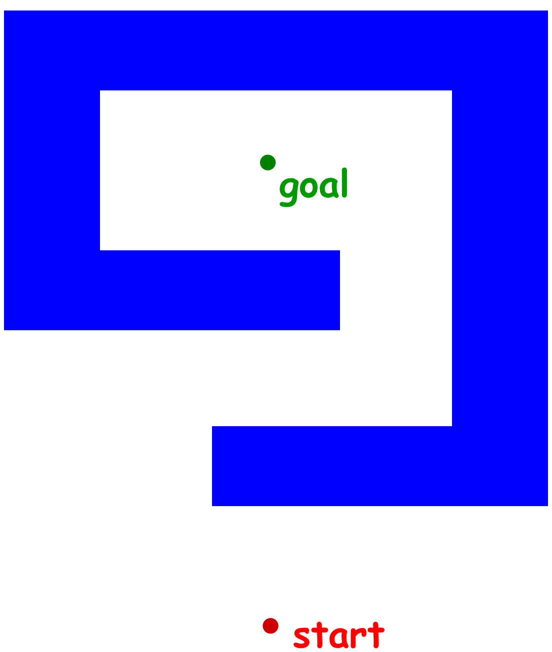
1) head toward goal

2) follow obstacles until you can head toward the goal again

3) continue

Bug Zapper

What map will foil Bug 0 ?



"Bug 0" algorithm

1) head toward goal

2) follow obstacles until you can head toward the goal again

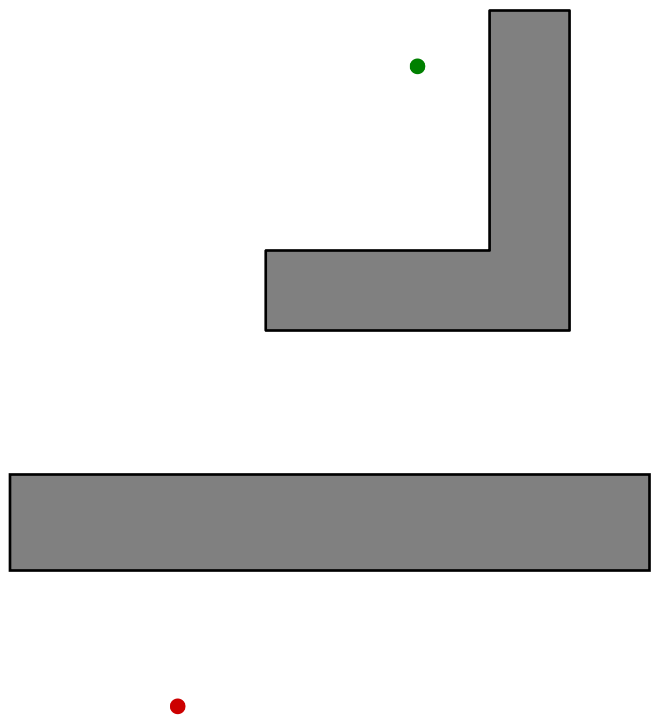
3) continue

A better bug?

But add <u>some</u> memory!

- known direction to goal
- otherwise local sensing

walls/obstacles & **encoders**



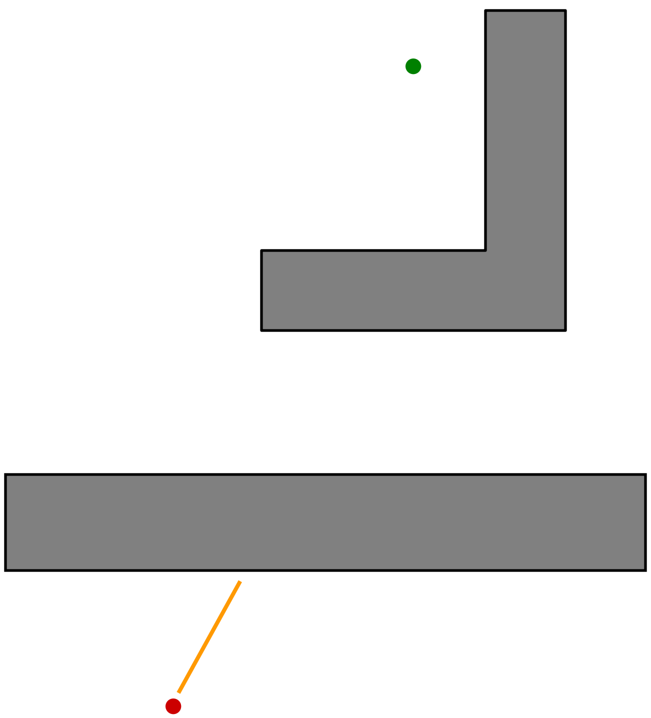
improvement ideas?

Bug 1

But some computing power!

- known direction to goal
- otherwise local sensing

walls/obstacles & **encoders**



"Bug 1" algorithm

1) head toward goal

2) if an obstacle is encountered, circumnavigate it *and* remember how close you get to the goal

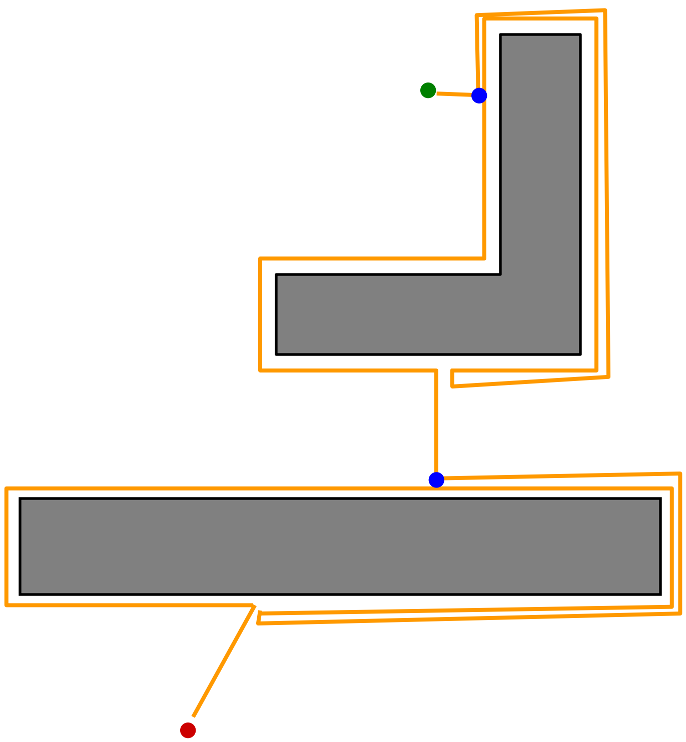
3) return to that closest point (by wall-following) and continue

Vladimir Lumelsky & Alexander Stepanov: Algorithmica 1987

Bug 1

But some computing power!
- known direction to goal
- otherwise local sensing

walls/obstacles & **encoders**



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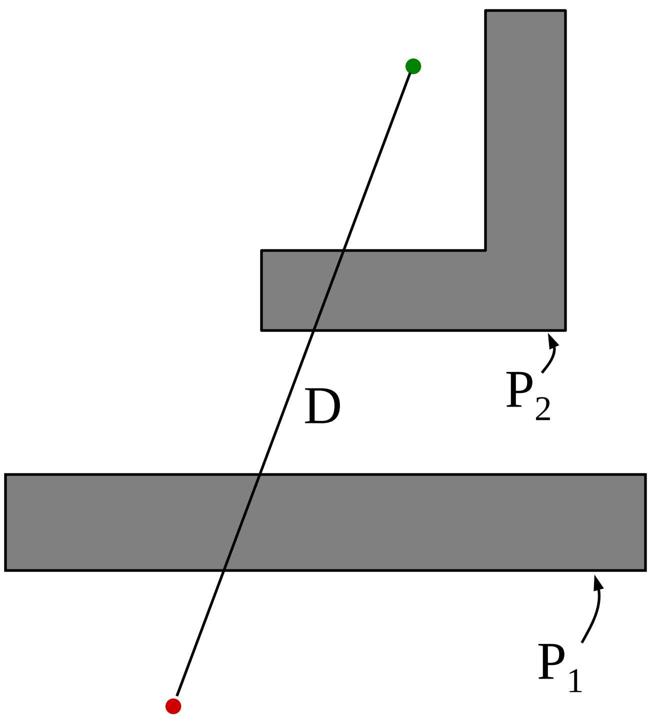
BUG 1 More formally

- Let $q^L_0 = q_{start}$; i = 1
- repeat
  - repeat
    - from q^L_{i-1} move toward q_{goal}
  - until goal is reached or obstacle encountered at q^H_i
  - if goal is reached, exit
  - repeat
    - follow boundary recording pt q^L_i with shortest distance to goal
  - until q_{goal} is reached or q^H_i is re-encountered
  - if goal is reached, exit
  - Go to q^L_i
  - if move toward q_{goal} moves into obstacle
    - exit with failure
  - else
    - i=i+1
    - continue

"Quiz" Bug 1 analysis

Bug 1: Path Bounds

What are upper/lower bounds on the path length that the robot takes?



D = straight-line distance from start to goal

P_i = perimeter of the *i* th obstacle

Lower bound:

What's the shortest distance it might travel?

Upper bound:

What's the longest distance it might travel?

What is an environment where your upper bound is <u>required</u>?

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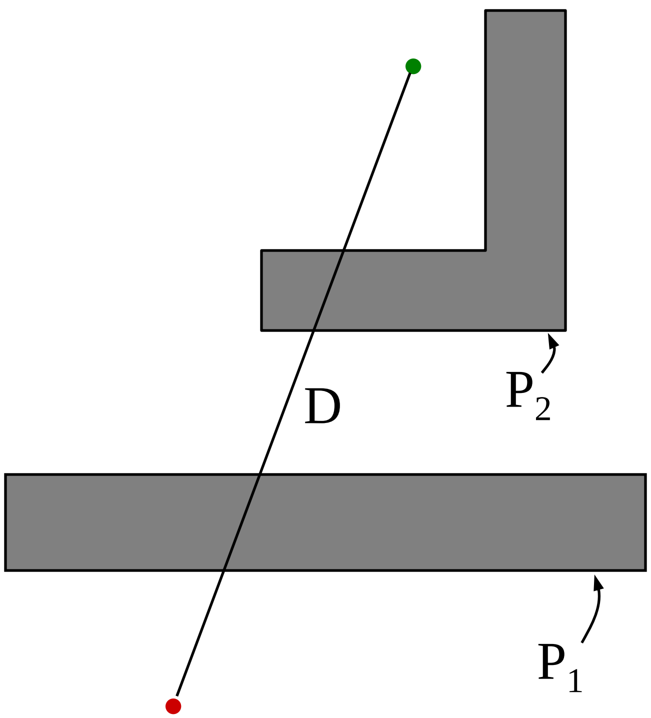


D = straight-line distance from start to goal

P_i = perimeter of the *i* th obstacle

Lower bound:
What's the shortest distance it might travel?

D

Upper bound:
What's the longest distance it might travel?

$$\mathbf{D + 1.5 \sum_i P_i}$$

What is an environment where your upper bound is required?

How Can We Show Completeness?

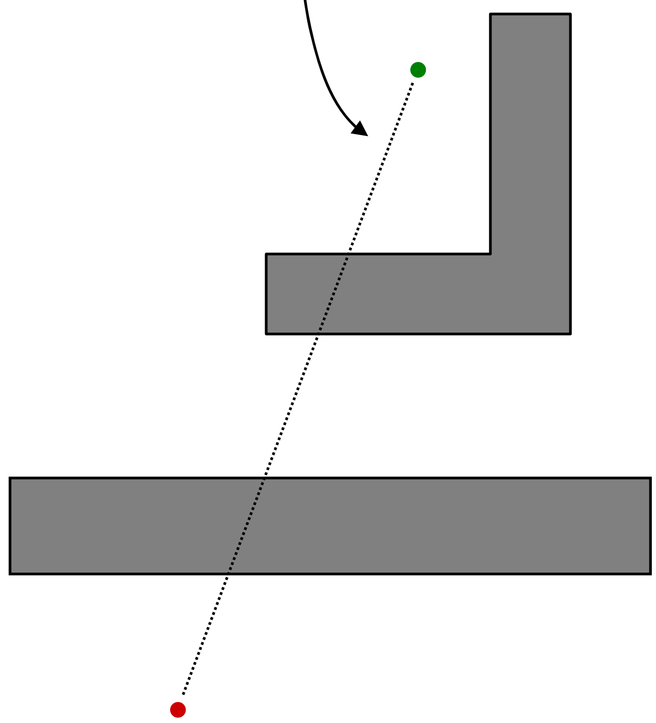
- An algorithm is *complete* if, in finite time, it finds a path if such a path exists or terminates with failure if it does not.

- Suppose BUG1 were incomplete
  - Therefore, there is a path from start to goal
    - By assumption, it is finite length, and intersects obstacles a finite number of times.
  - BUG1 does not find it
    - Either it terminates incorrectly, or, it spends an infinite amount of time
    - Suppose it never terminates
      - but each leave point is closer to the obstacle than corresponding hit point
      - Each hit point is closer than the last leave point
      - Thus, there are a finite number of hit/leave pairs; after exhausting them, the robot will proceed to the goal and terminate
    - Suppose it terminates (incorrectly)
    - Then, the closest point after a hit must be a leave where it would have to move into the obstacle
      - But, then line from robot to goal must intersect object even number of times (Jordan curve theorem)
      - But then there is another intersection point on the boundary closer to object. Since we assumed there is a path, we must have crossed this pt on boundary which contradicts the definition of a leave point.

Another step forward?

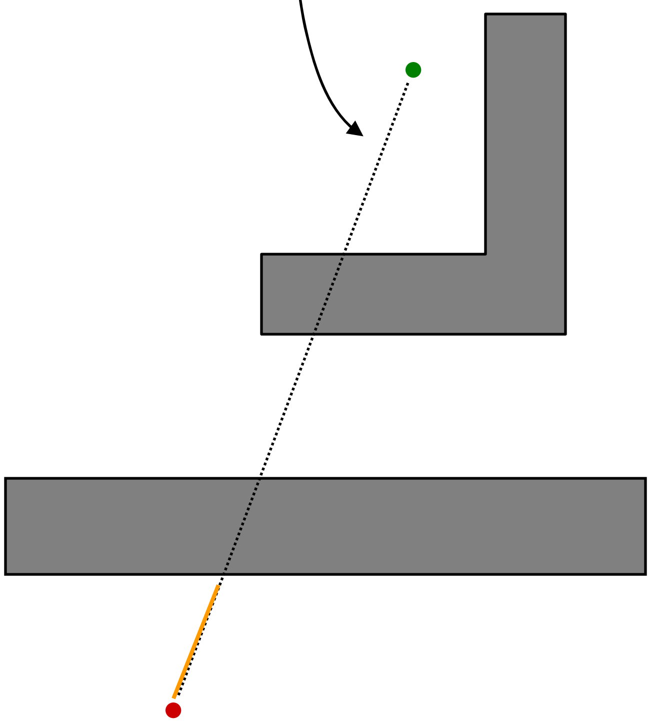
Call the line from the starting point to the goal the ***m-line***

"Bug 2" Algorithm



A better bug?

Call the line from the starting point to the goal the ***m-line***

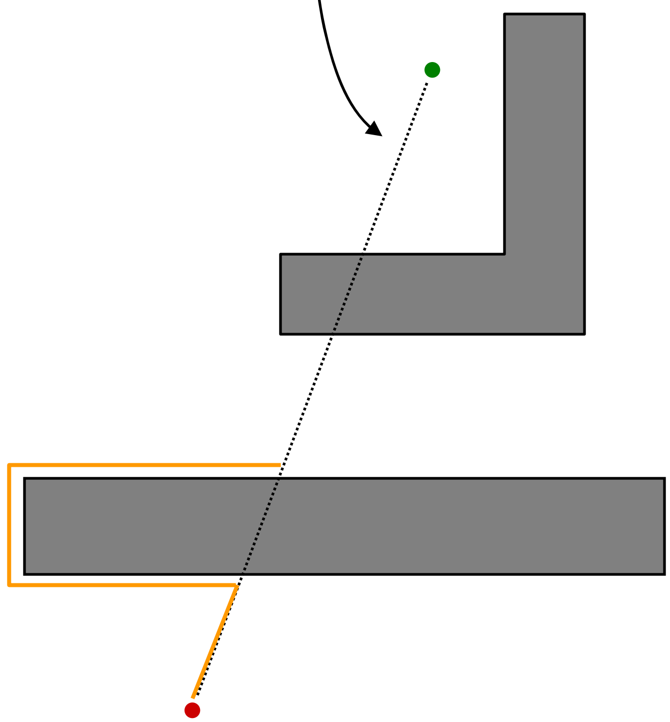


"Bug 2" Algorithm

1) head toward goal on the *m-line*

A better bug?

Call the line from the starting point to the goal the ***m-line***

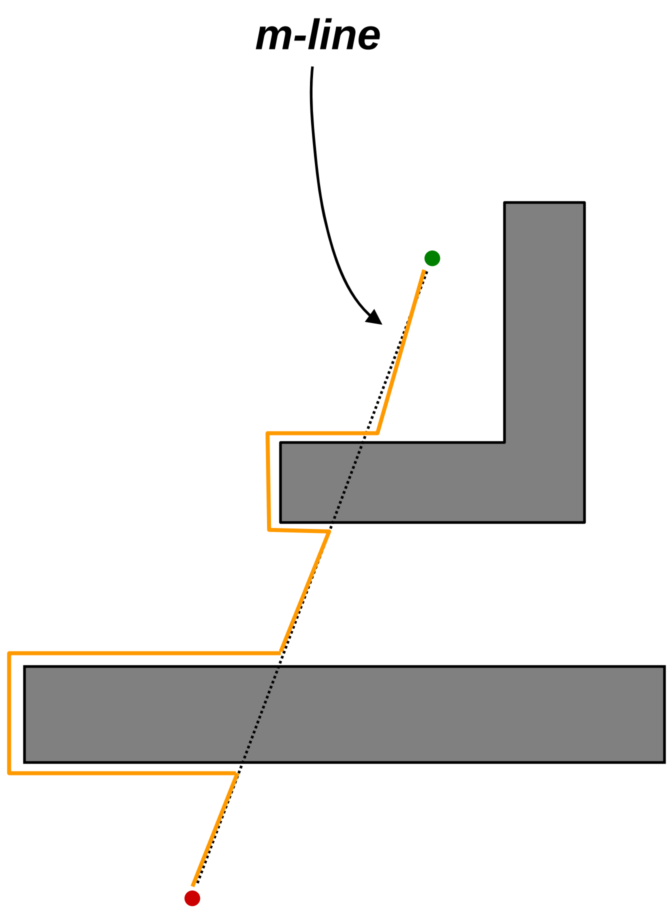


"Bug 2" Algorithm

1) head toward goal on the *m-line*

2) if an obstacle is in the way, follow it until you encounter the m-line again.

A better bug?



"Bug 2" Algorithm

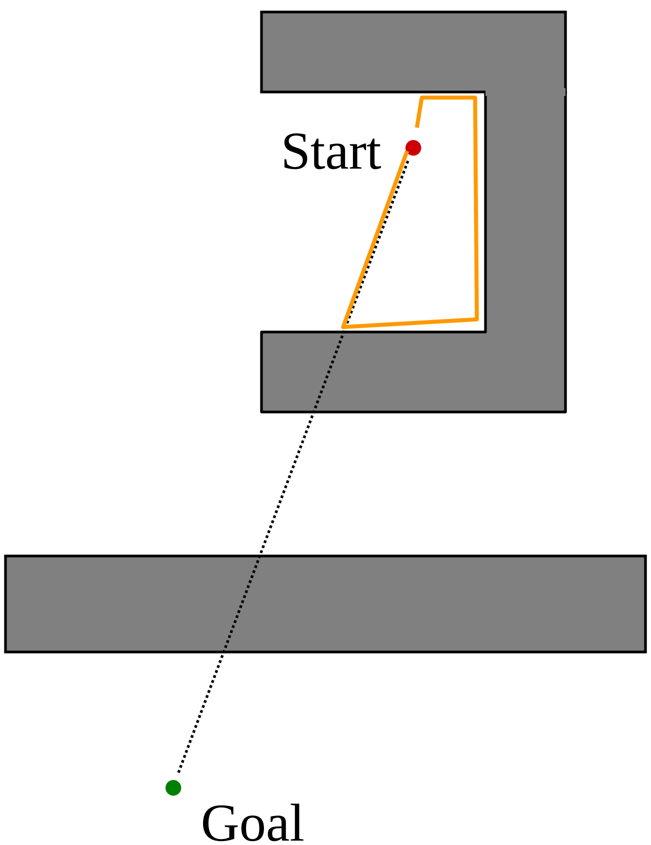
1) head toward goal on the *m-line*

2) if an obstacle is in the way, follow it until you encounter the m-line again.

3) Leave the obstacle and continue toward the goal

OK ?

A better bug?



"Bug 2" Algorithm

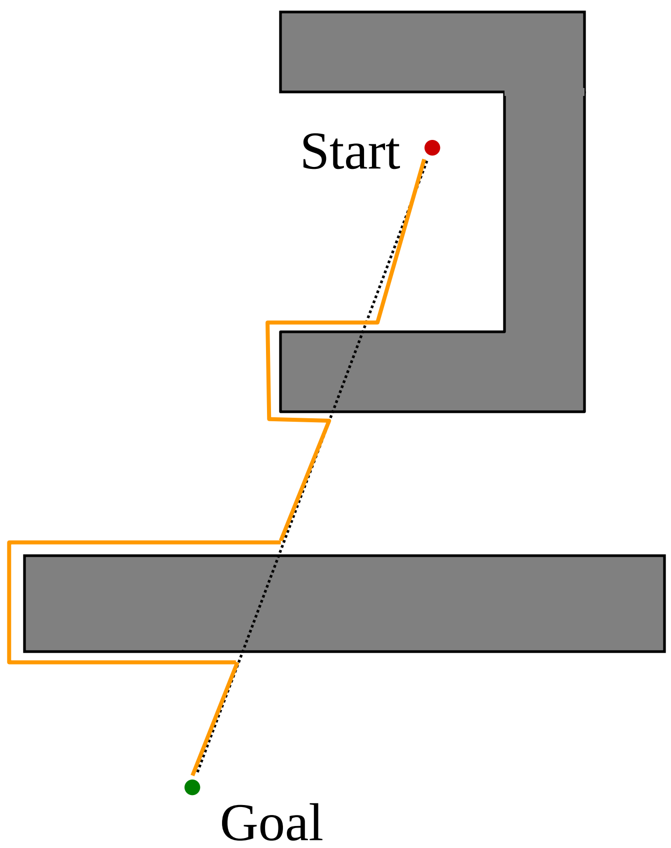
1) head toward goal on the *m-line*

2) if an obstacle is in the way, follow it until you encounter the m-line again.

3) Leave the obstacle and continue toward the goal

NO! How do we fix this?

A better bug?



"Bug 2" Algorithm

1) head toward goal on the *m-line*

2) if an obstacle is in the way, follow it until you encounter the m-line again ***closer to the goal***.

3) Leave the obstacle and continue toward the goal

Better or worse than Bug1?

BUG 2 More formally

- Let $q^L_0 = q_{start;}$ i = 1
- repeat
  – repeat
    - from q^L_{i-1} move toward q_{goal} along the m-line
  – until goal is reached or obstacle encountered at q^H_i
  – if goal is reached, exit
  – repeat
    - follow boundary
  – until q_{goal} is reached or q^H_i is re-encountered or m-line is re-encountered, x is not q^H_i, $d(x,q_{goal}) < d(q^H_i,q_{goal})$ and way to goal is unimpeded
  – if goal is reached, exit
  – if q^H_i is reached, return failure
  – else
    - $q^L_i = m$
    - i=i+1
    - continue

head-to-head comparison

or thorax-to-thorax, perhaps

Draw worlds in which Bug 2 does better than Bug 1 (and vice versa).

Bug 2 beats Bug 1	Bug 1 beats Bug 2

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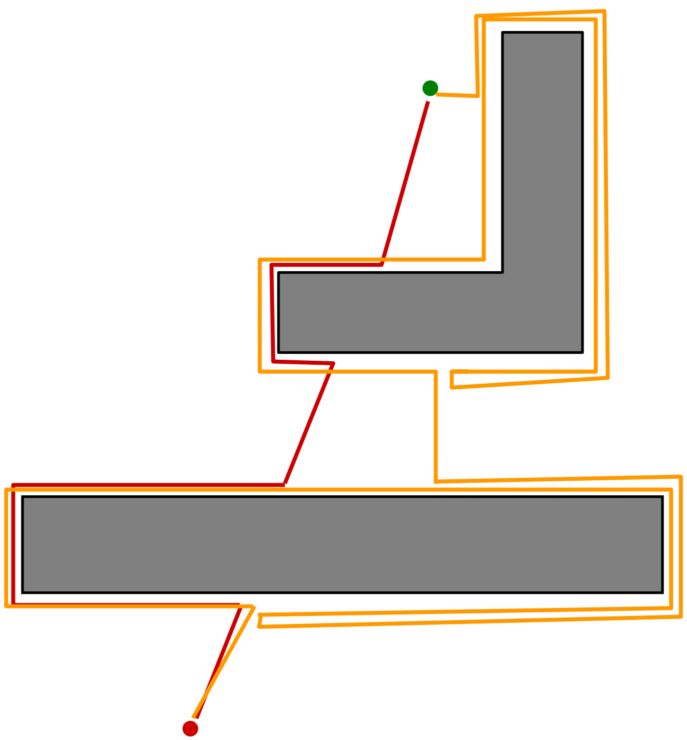
Bug 2 beats Bug 1



Bug 1 beats Bug 2

?

head-to-head comparison

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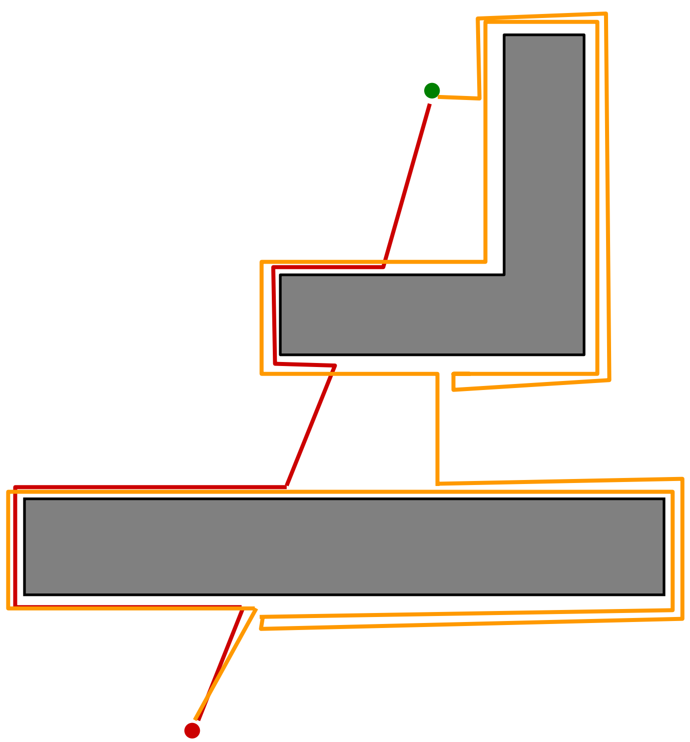
Draw worlds in which Bug 2 does better than Bug 1 (and vice versa).

Bug 2 beats Bug 1



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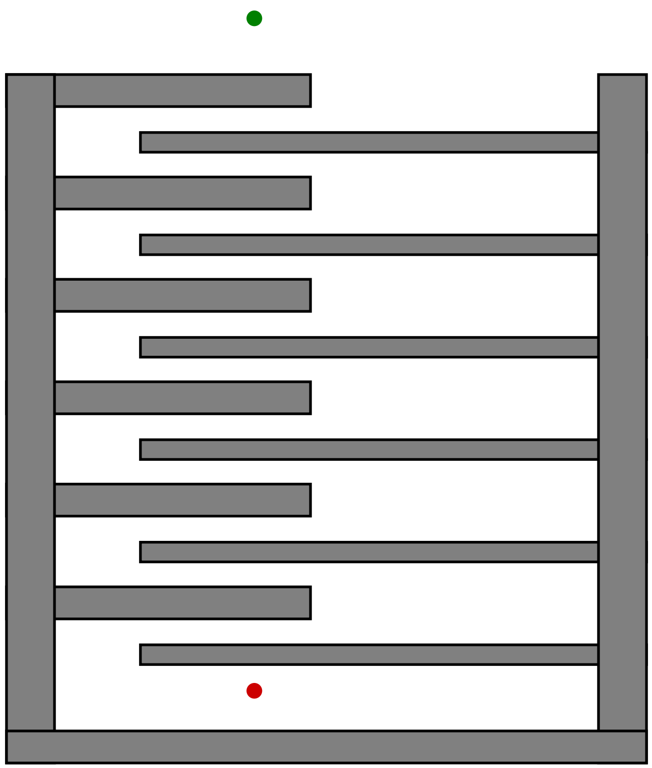

BUG 1 vs. BUG 2

- BUG 1 is an *exhaustive search algorithm*
  - it looks at all choices before commiting
- BUG 2 is a *greedy* algorithm
  - it takes the first thing that looks better
- In many cases, BUG 2 will outperform BUG 1, but
- BUG 1 has a more predictable performance overall

"Quiz" Bug 2 analysis

Bug 2: Path Bounds

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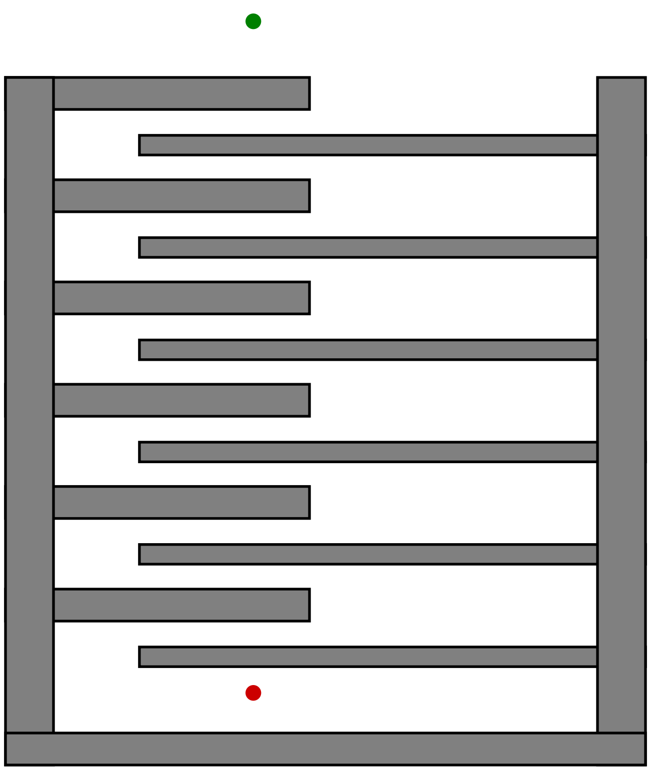

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P_i = perimeter of the *i* th obstacle

Lower bound:
What's the shortest distance it might travel?

D

Upper bound:
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What is an environment where your upper bound is required?

"Quiz" Bug 2 analysis

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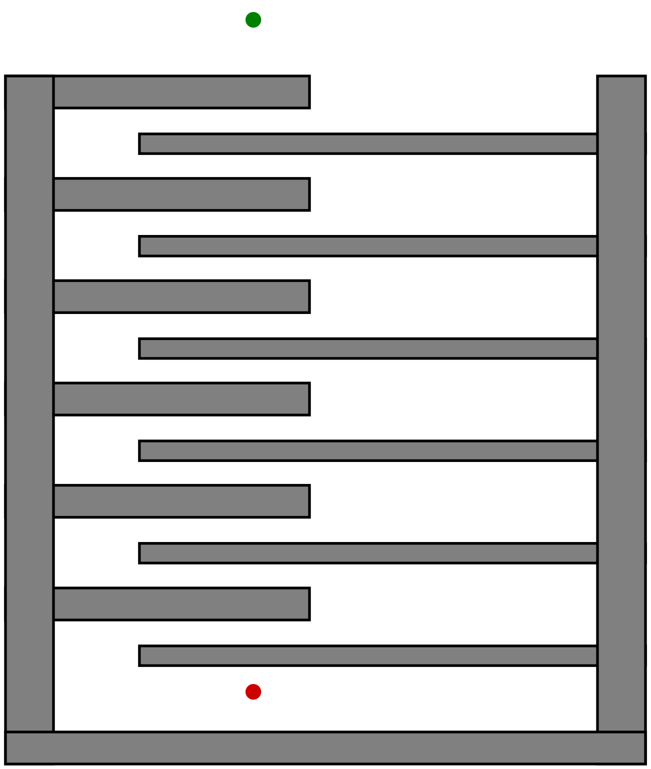

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Lower bound: What's the shortest distance it might travel?

$$D$$

Upper bound: What's the longest distance it might travel?

$$D + \sum_i \frac{n_i}{2} P_i$$

n_i = # of s-line intersections of the *i* th obstacle

What is an environment where your upper bound is required?

"Quiz" Bug 2 analysis

Bug 2: Path Bounds

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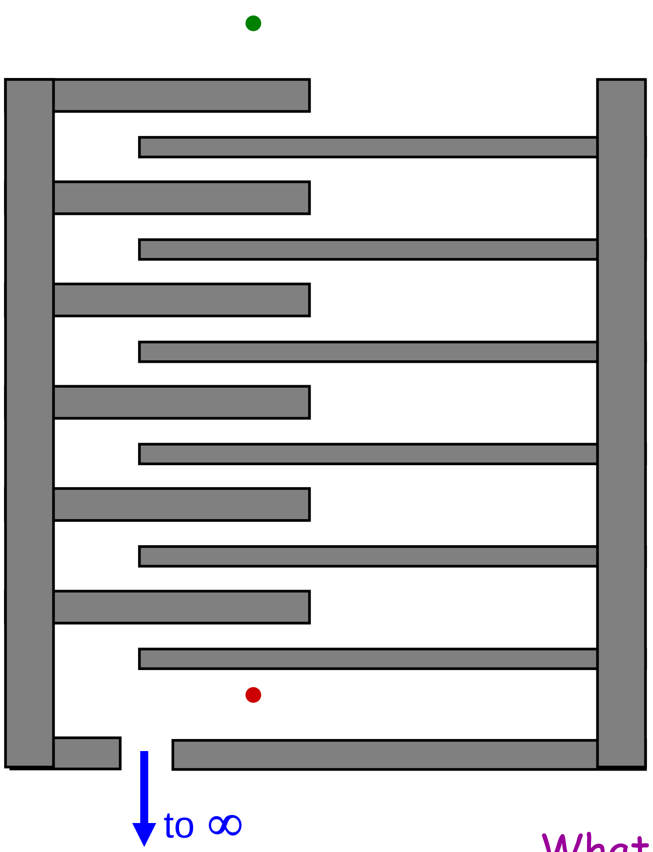

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What is an environment where your upper bound is required?

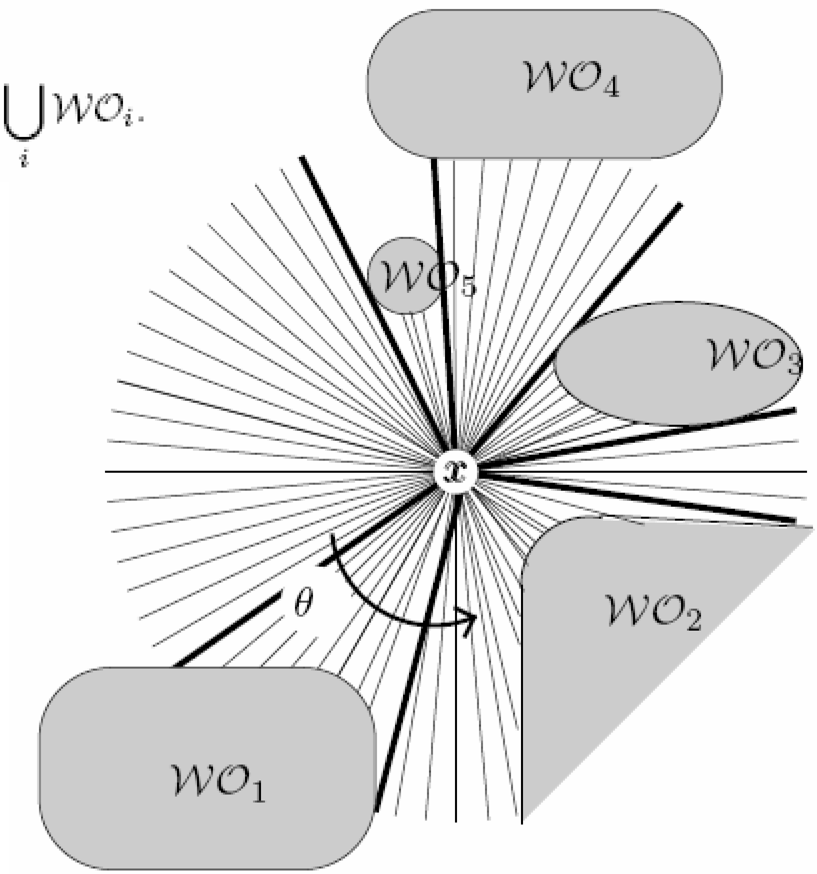
A More Realistic Bug

- As presented: global beacons plus contact-based wall following
- The reality: we typically use some sort of range sensing device that lets us look ahead (but has finite resolution and is noisy).
- Let us assume we have a range sensor

Raw Distance Function

$$\rho(x,\theta) = \min_{\lambda \in [0,\infty]} d(x, x + \lambda[\cos\theta, \sin\theta]^T),$$

$$\text{such that} \quad x + \lambda[\cos\theta, \sin\theta]^T \in \bigcup_i \mathcal{WO}_i.$$



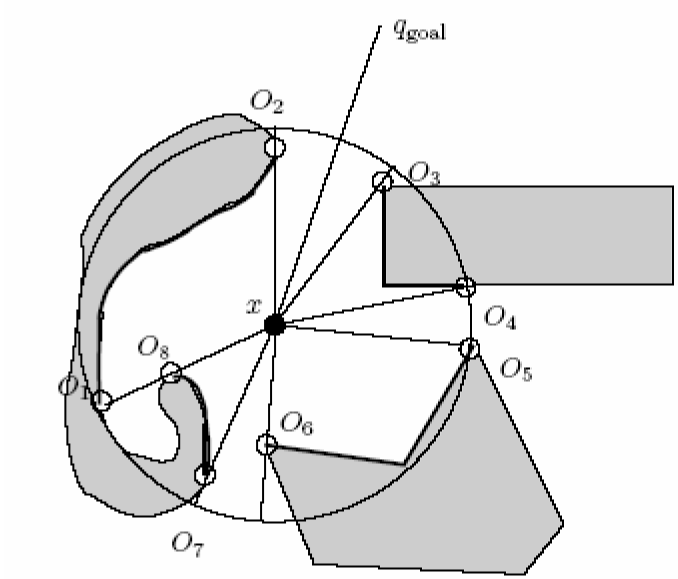
$$\rho : \mathbb{R}^2 \times S^1 \rightarrow \mathbb{R}$$

Saturated raw distance function

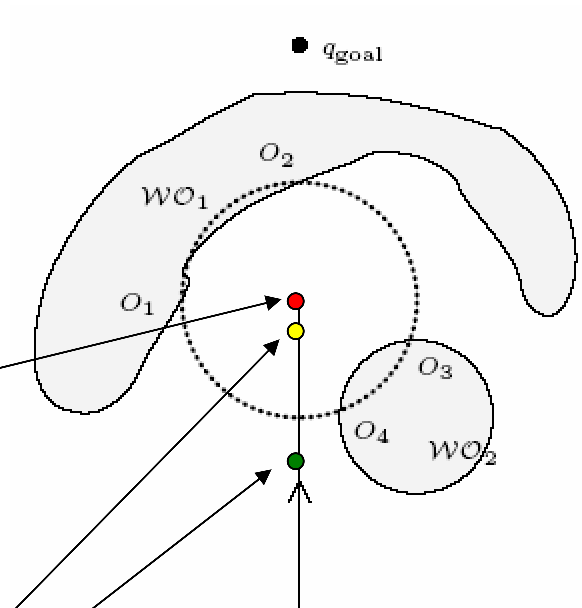
$$\rho_R(x,\theta) = \begin{cases} \rho(x,\theta), & \text{if } \rho(x,\theta) < R \\ \infty, & \text{otherwise.} \end{cases}$$

Intervals of Continuity

- Tangent Bug relies on finding endpoints of finite, conts segments of ρ_R



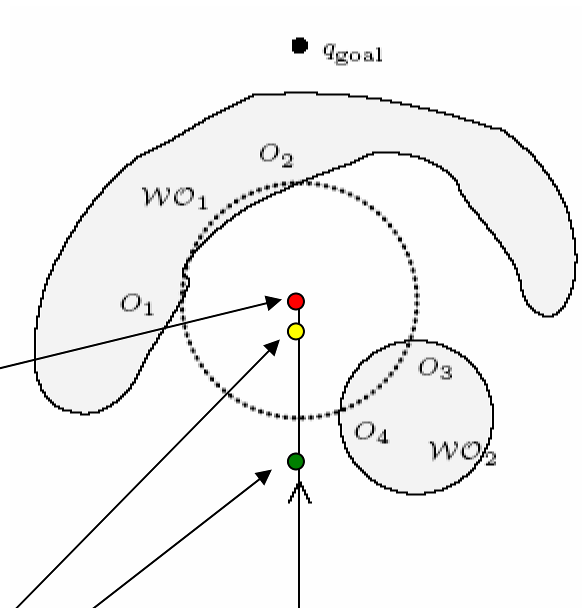
Motion-to-Goal Transition
from Moving Toward goal to “following obstalces”



Transition

Currently, the motion-to-goal behavior “thinks” the robot can get to the goal

Motion-to-Goal Transition
from Moving Toward goal to “following obstalces”



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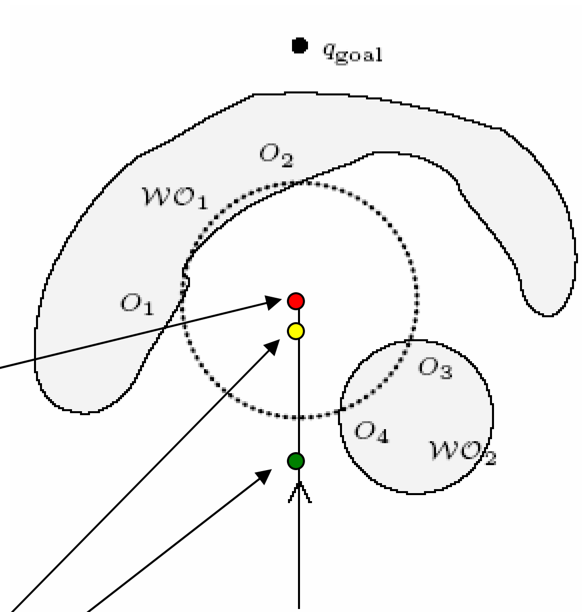
Motion-to-Goal Transition
Among Moving Toward goal to “following obstacles”



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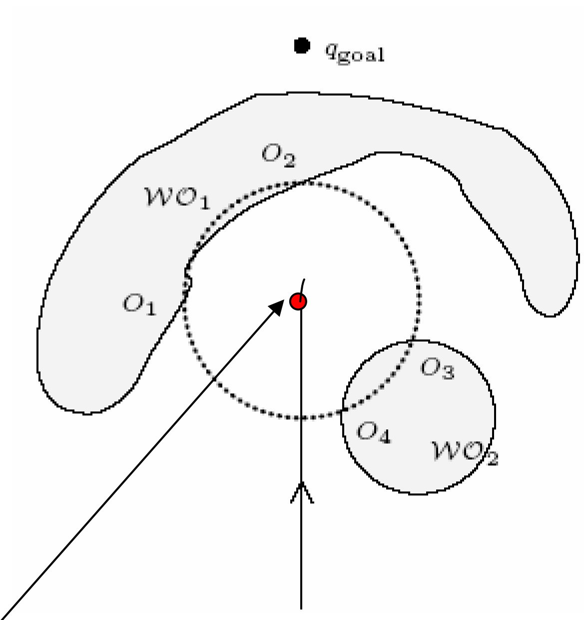
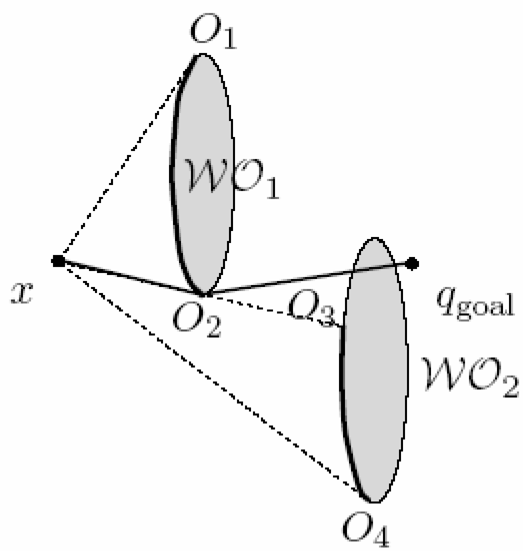
Motion-to-Goal Transition
Minimize Heuristic



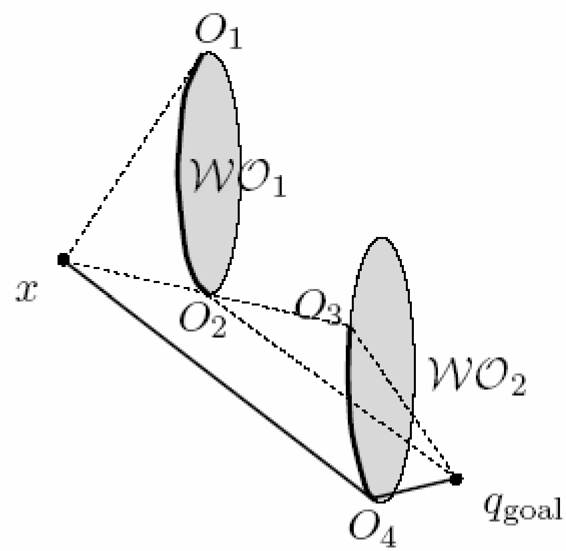
Now, it starts to see something --- what to do?
Ans: Choose the pt O_i that minimizes $d(x,O_i) + d(O_i,q_{goal})$

Minimize Heuristic Example

At x, robot knows only what it sees and where the goal is,



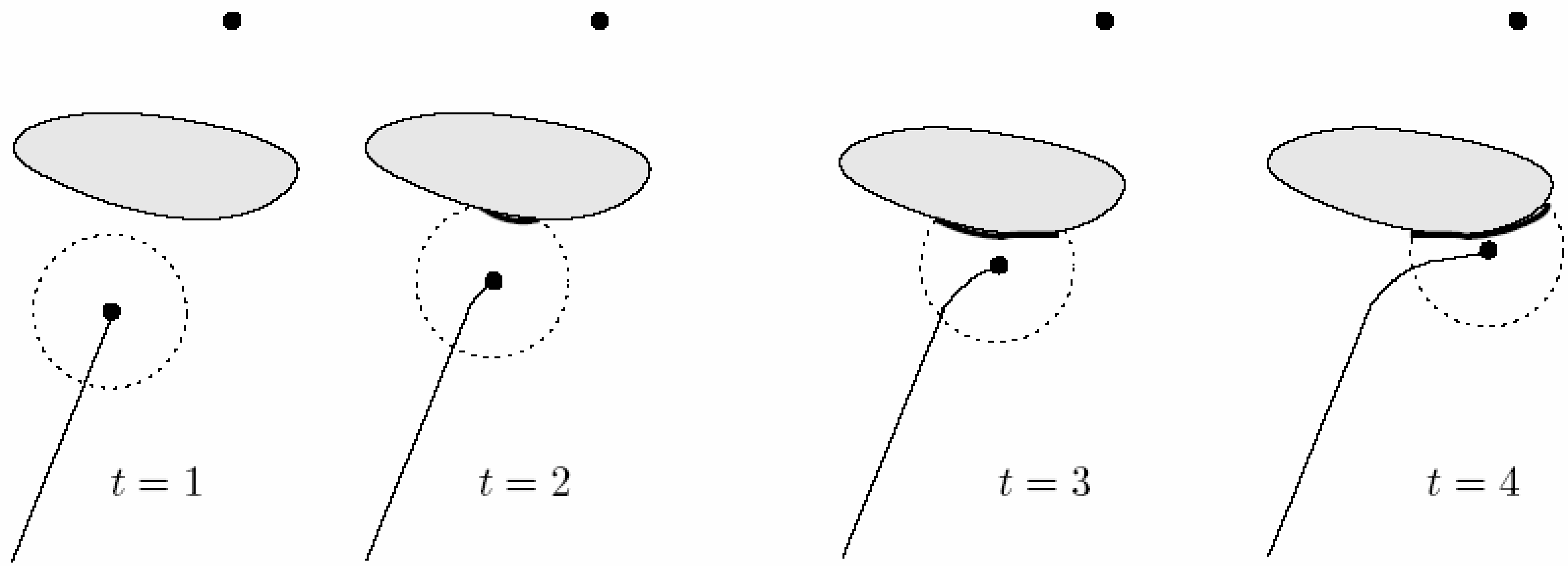
so moves toward O_2. Note the line connecting O_2 and goal pass through obstacle



so moves toward O_4. Note some "thinking" was involved and the line connecting O_4 and goal pass through obstacle

Choose the pt O_i that minimizes $d(x,O_i) + d(O_i,q_{goal})$

Motion To Goal Example



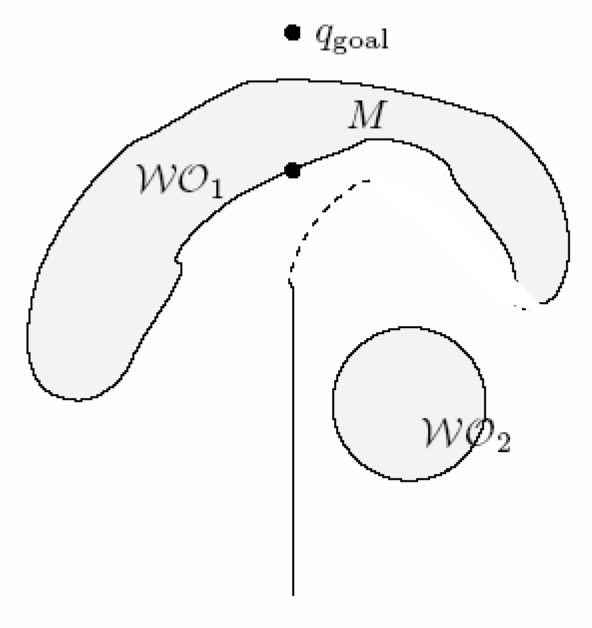
Choose the pt O_i that minimizes $d(x,O_i) + d(O_i,q_{goal})$

Transition *from* Motion-to-Goal

Choose the pt O_i that minimizes $d(x,O_i) + d(O_i,q_{goal})$

Problem: what if this distance starts to go up?

Ans: start to act like a BUG and follow boundary



M is the point on the "sensed" obstacle which has the shorted distance to the goal

Followed obstacle: the obstacle that we are currently sensing

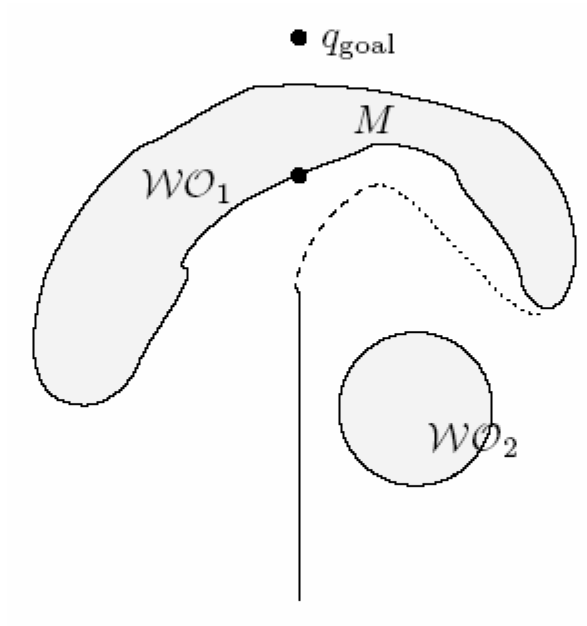
Blocking obstacle: the obstacle that intersects the segment

$$(1-\lambda)x + \lambda q_{\text{goal}} \quad \forall \lambda \in [0,1]$$

They start as the same

Boundary Following

Move toward the O_i on the followed obstacle in the "chosen" direction



M is the point on the "sensed" obstacle which has the shorted distance to the goal

Followed obstacle: the obstacle that we are currently sensing

Blocking obstacle: the obstacle that intersects the segment

Maintain d_{followed} and d_{reach}

They start as the same

$d_{followed}$ and d_{reach}

- $d_{followed}$ is the shortest distance between the sensed boundary and the goal

- d_{reach} is the shortest distance between *blocking* obstacle and goal (or my distance to goal if no blocking obstacle visible)

$$\Lambda = \{y \in \partial \mathcal{WO}_b : \lambda x + (1-\lambda) y \in \mathcal{Q}_{\text{free}} \quad \forall \lambda \in [0,1]\}.$$

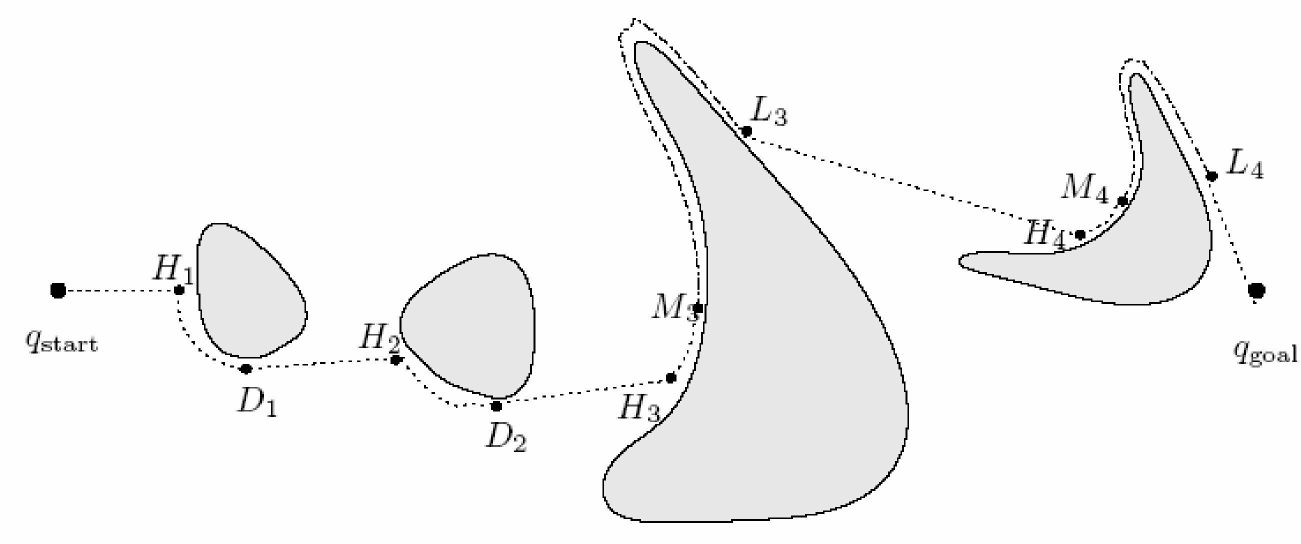
$$d_{\text{reach}} = \min_{c \in \Lambda} d(q_{\text{goal}}, c)$$

- Terminate boundary following behavior when $d_{reach} < d_{followed}$

- Initialize with $x = q_{\text{start}}$ and $d_{\text{leave}} = d(q_{\text{start}}, q_{\text{goal}})$

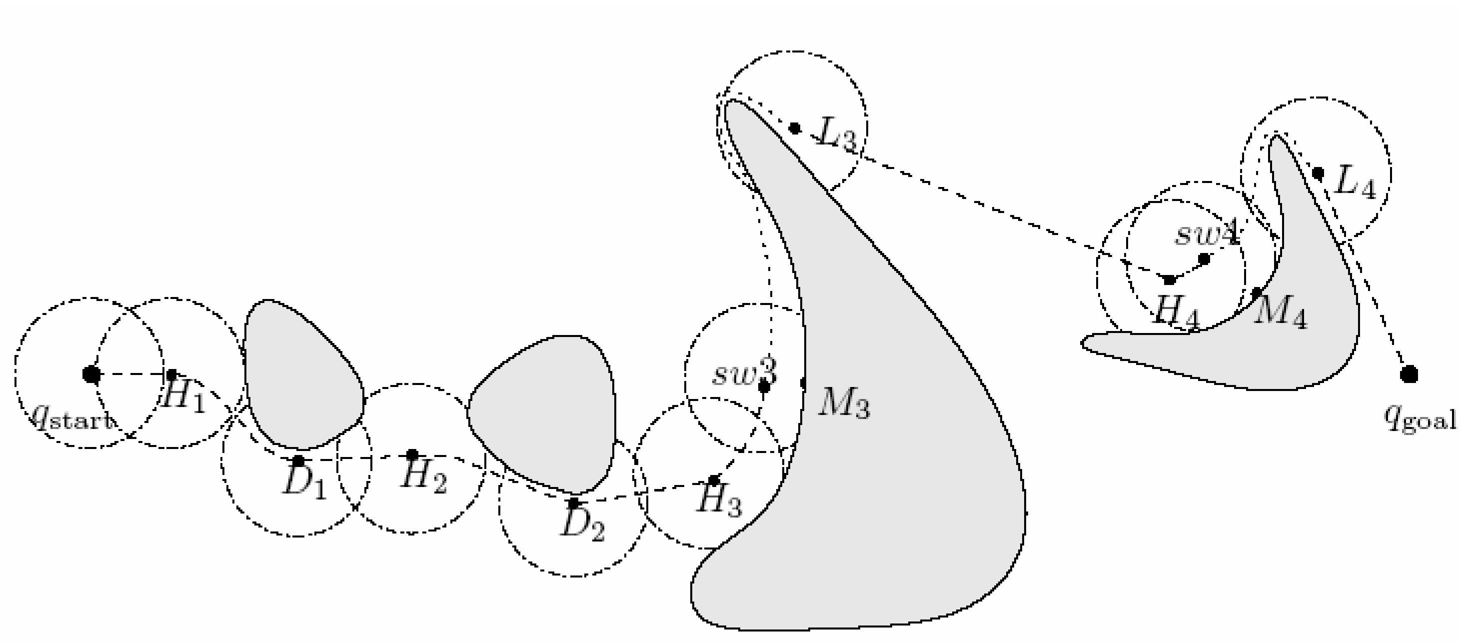
typo

Example: Zero Senor Range

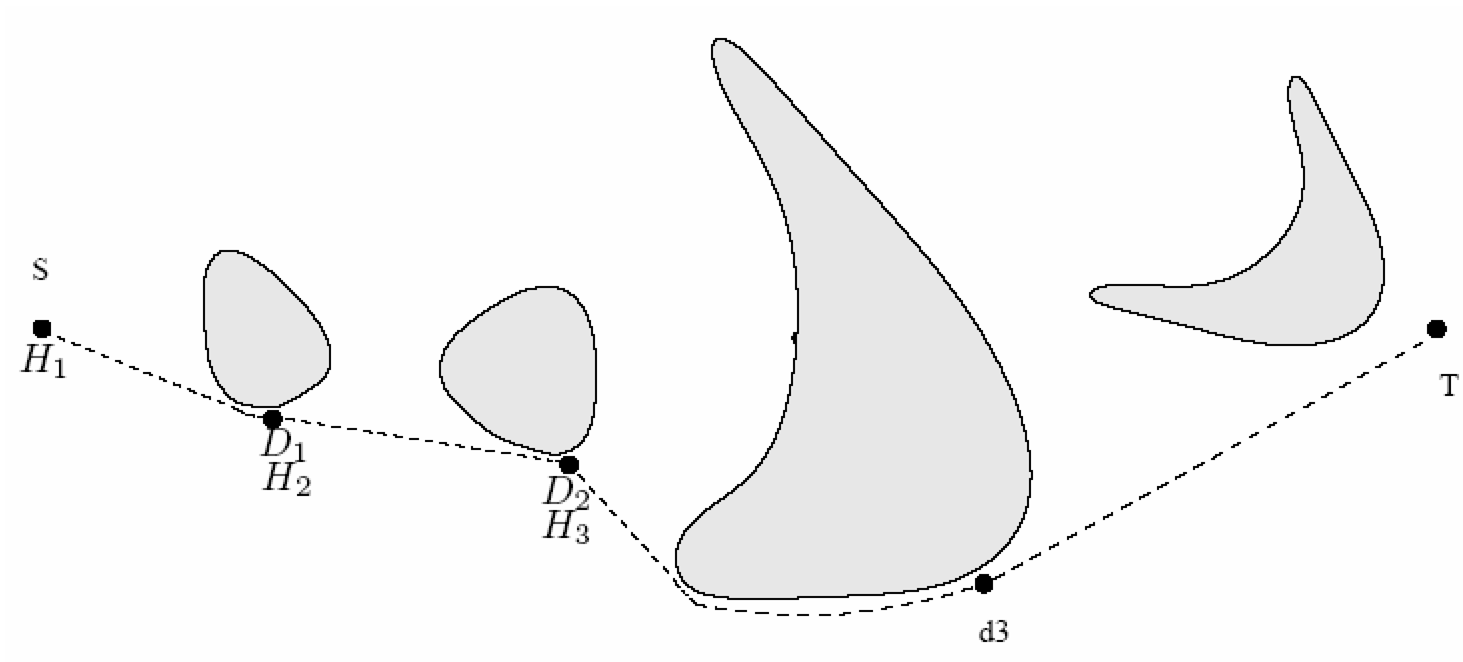


1. Robot moves toward goal until it hits obstacle 1 at H1
2. Pretend there is an infinitely small sensor range and the Oi which minimizes the heuristic is to the right
3. Keep following obstacle until robot can go toward obstacle again
4. Same situation with second obstacle
5. At third obstacle, the robot turned left until it could not increase heuristic
6. $D_{followed}$ is distance between M_3 and goal, d_{reach} is distance between robot and goal because sensing distance is zero

Example: Finite Sensor Range

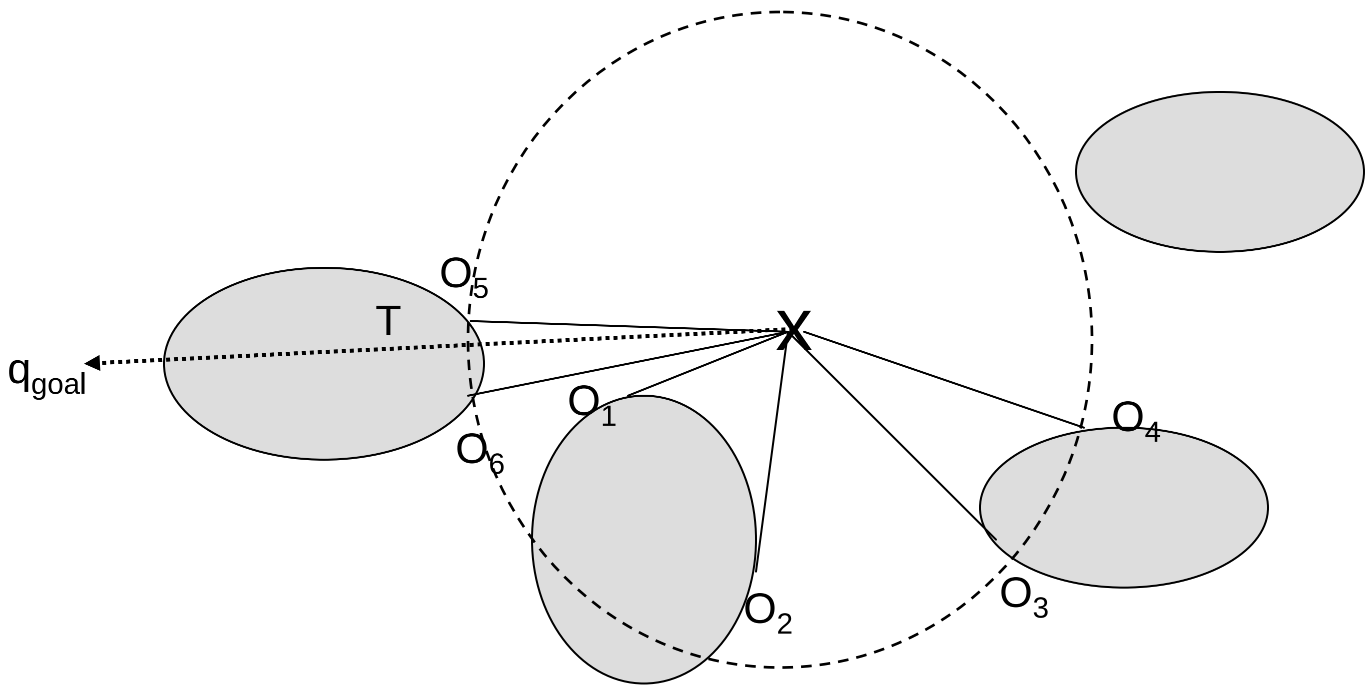


Example: Infinite Sensor Range



Tangent Bug

- Tangent Bug relies on finding endpoints of finite, conts segments of ρ_R

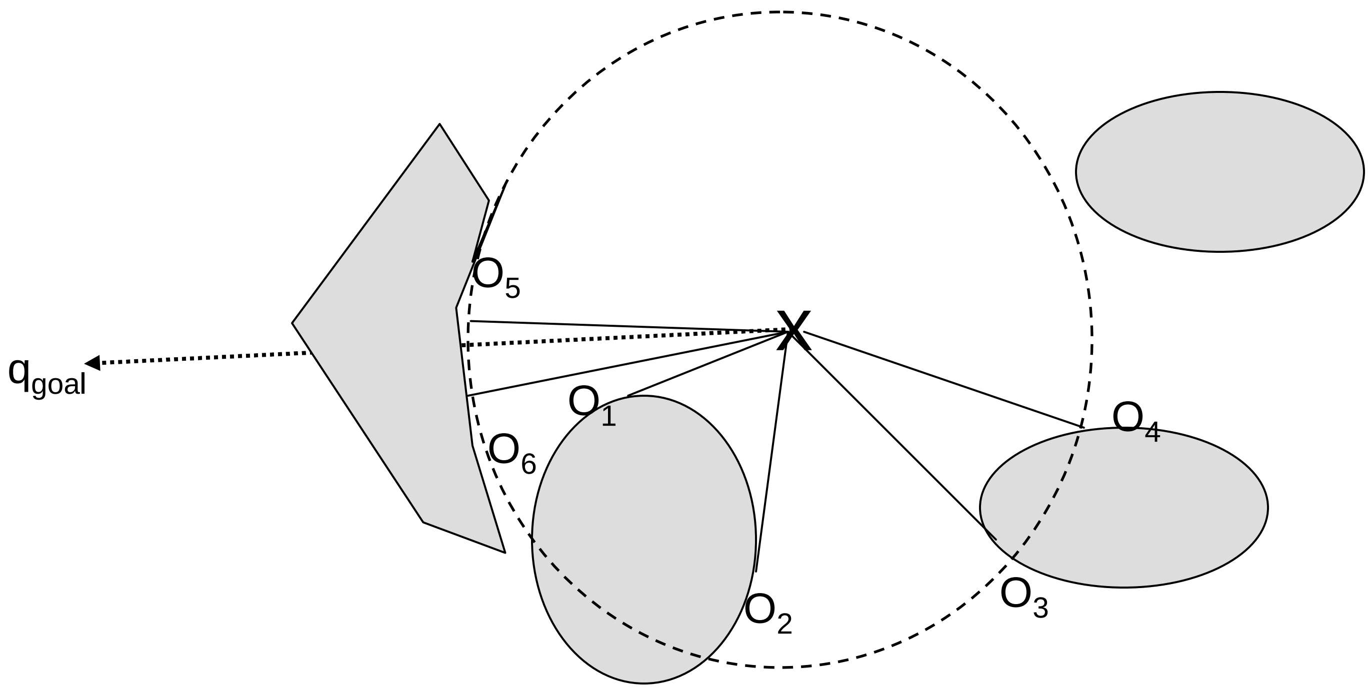


Now, it starts to see something --- what to do?
Ans: Choose the pt O_i that minimizes $d(x,O_i) + d(O_i,q_{goal})$

"Heuristic distance"

Tangent Bug

- Tangent Bug relies on finding endpoints of finite, conts segments of ρ_R



Problem: what if this distance starts to go up?
Ans: start to act like a BUG and follow boundary

The Basic Ideas

- A motion-to-goal behavior as long as way is clear or there is a visible obstacle boundary pt that decreases heuristic distance
- A boundary following behavior invoked when heuristic distance increases.
- A value $d_{followed}$ which is the shortest distance between the sensed boundary and the goal
- A value d_{reach} which is the shortest distance between *blocking* obstacle and goal (or my distance to goal if no blocking obstacle visible)
- Terminate boundary following behavior when $d_{reach} < d_{followed}$

Tangent Bug Algorithm

1) repeat
   a) Compute continuous range segments in view
   b) Move toward $n \in \{T, O_i\}$ that minimizes $h(x,n) = d(x,n) + d(n,q_{goal})$

   until
   a) goal is encountered, or
   b) the value of $h(x,n)$ begins to increase

2) follow boundary continuing in same direction as before repeating
   a) update $\{O_i\}$, d_{reach} and $d_{followed}$

   until
   a) goal is reached
   b) a complete cycle is performed (goal is unreachable)
   c) $d_{reach} < d_{followed}$

Note the same general proof reasoning as before applies, although the definition of hit and leave points is a little trickier.

Implementing Tangent Bug

- Basic problem: compute tangent to curve forming boundary of obstacle at any point, and drive the robot in that direction

- Let $D(x) = \min_c d(x,c) \quad c \in \cup_i WO_i$
- Let $G(x) = D(x) - W^*$ $\leftarrow$ some safe following distance
- Note that $\nabla G(x)$ points radially away from the object
- Define $T(x) = (\nabla G(x))$ the tangent direction
  - in a real sensor (we'll talk about these) this is just the tangent to the array element with lowest reading

- We could just move in the direction T(x)
  - open-loop control

- Better is $\delta x = \mu (T(x) - \lambda (\nabla G(x)) G(x))$
  - closed-loop control (predictor-corrector)

Sensors!

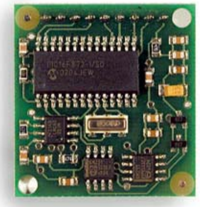
Robots' link to the external world...

sonar rangefinder

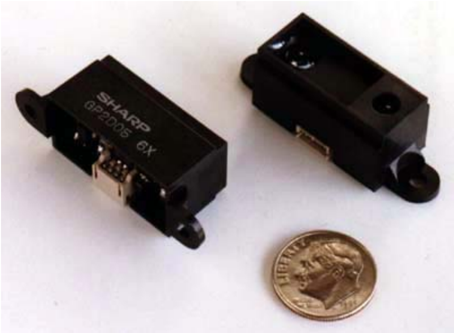
gyro

Sensors, sensors, sensors!

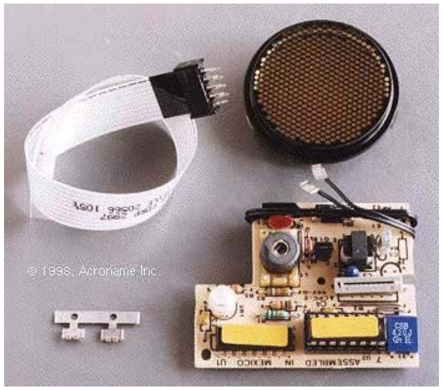
and tracking what is sensed: world models



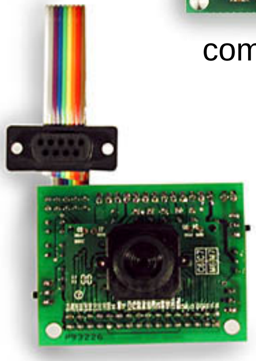
compass



IR rangefinder



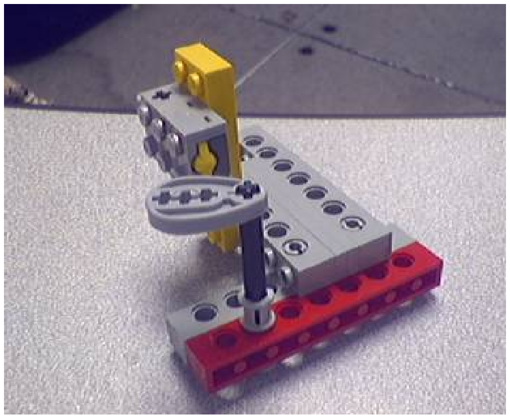
sonar rangefinder



CMU cam with on-board processing

odometry…

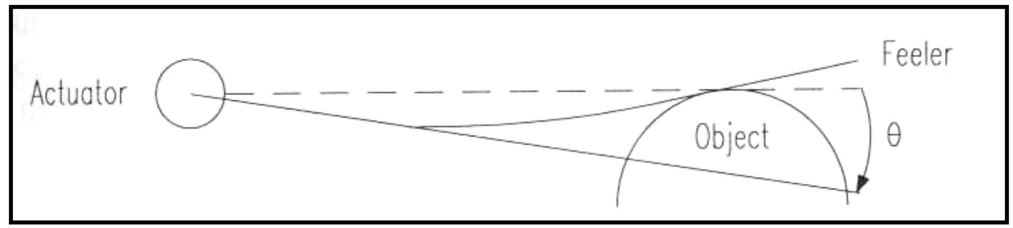
Tactile sensors

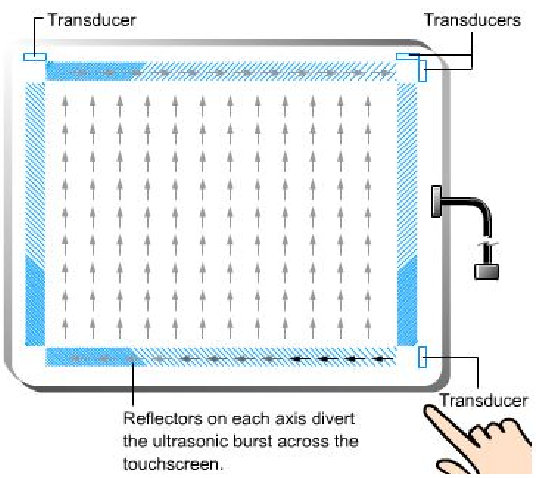


on/off switch

as a low-resolution encoder…

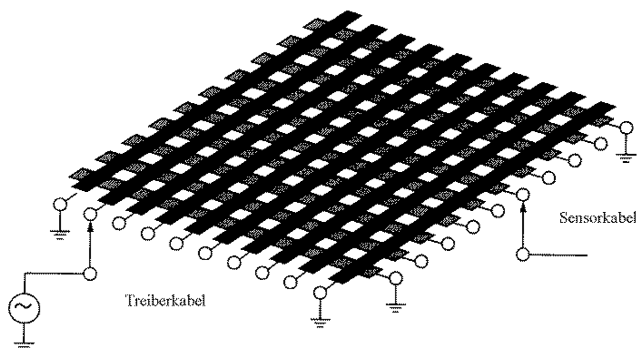
analog input: “Active antenna”





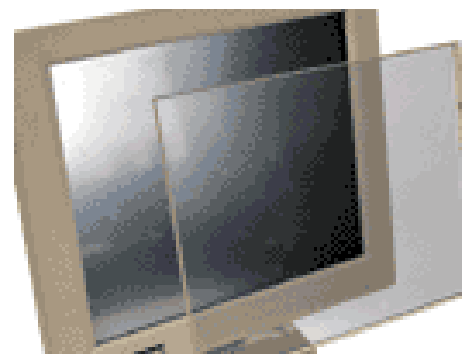
Surface acoustic waves

100% of light passes through



Capacitive array sensors

90% of light passes through



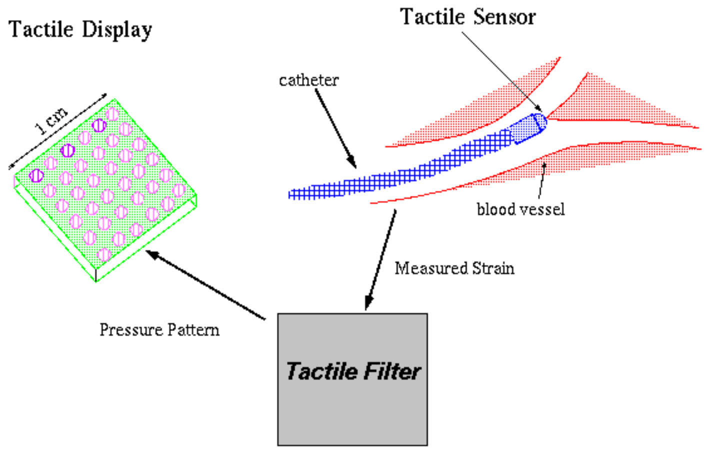
Resistive sensors

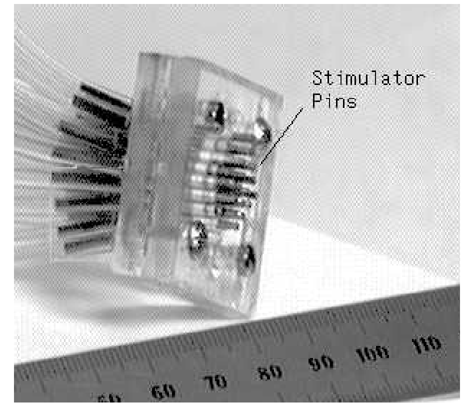
75% of light passes through

Tactile applications

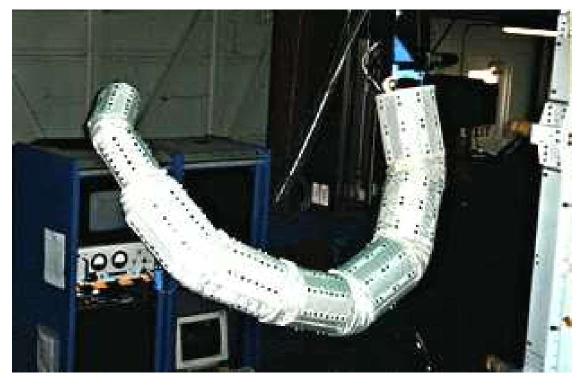
Medical teletaction interfaces

daVinci medical system





haptics

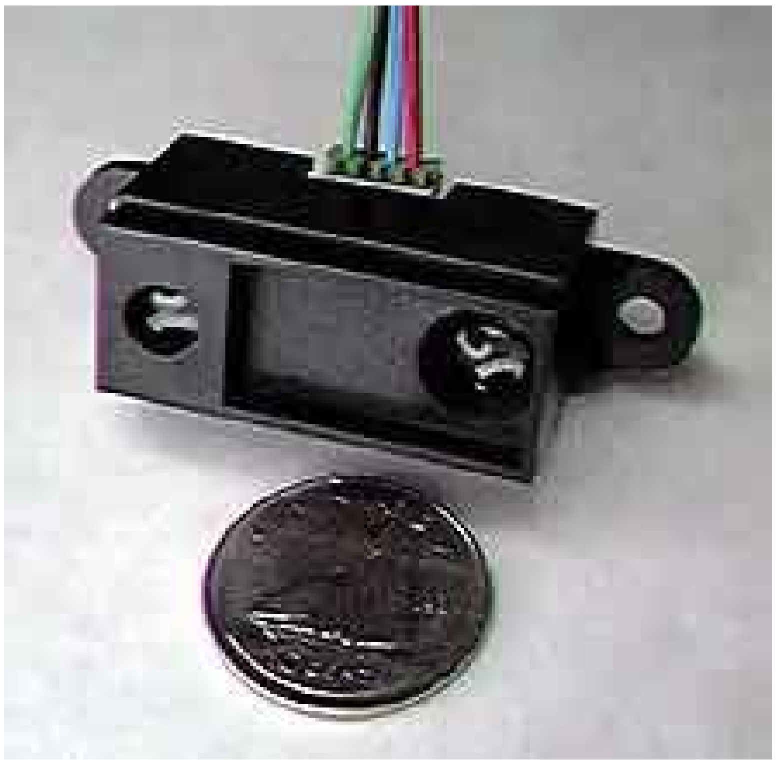


Robotic sensing

Merritt systems, FL

Infrared sensors

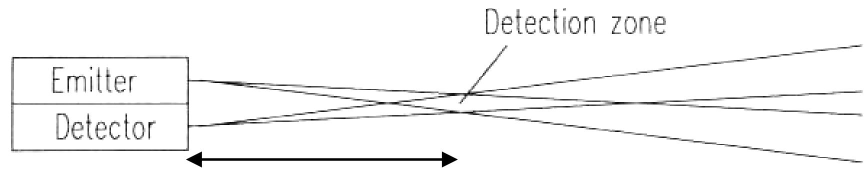
“Noncontact bump sensor”



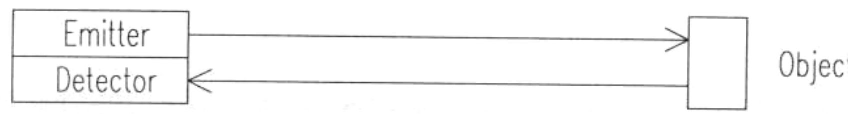
IR emitter/detector pair

(1) sensing is based on light intensity.

“object-sensing” IR

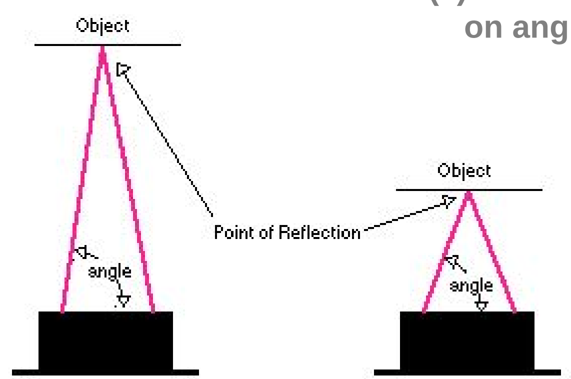


looks for changes
at this distance



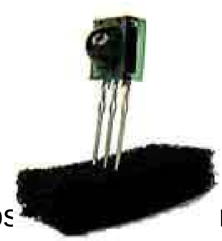
diffuse distance-sensing IR

(2) sensing is based
on angle receved.



Different Angles with Different Distances

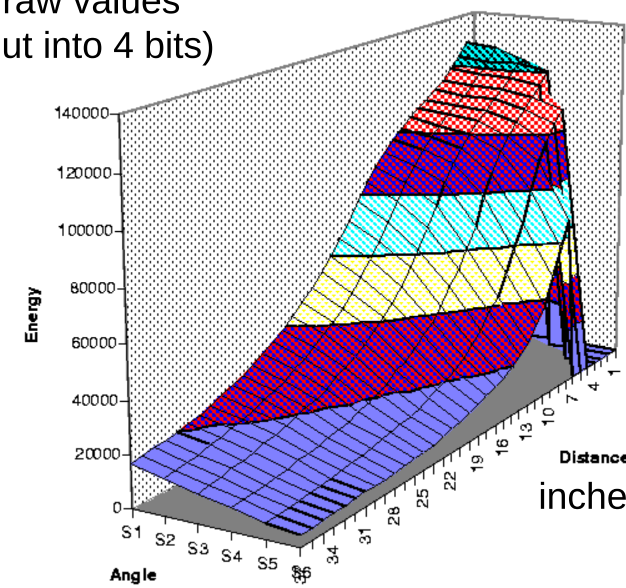
IR detector



Infrared calibration

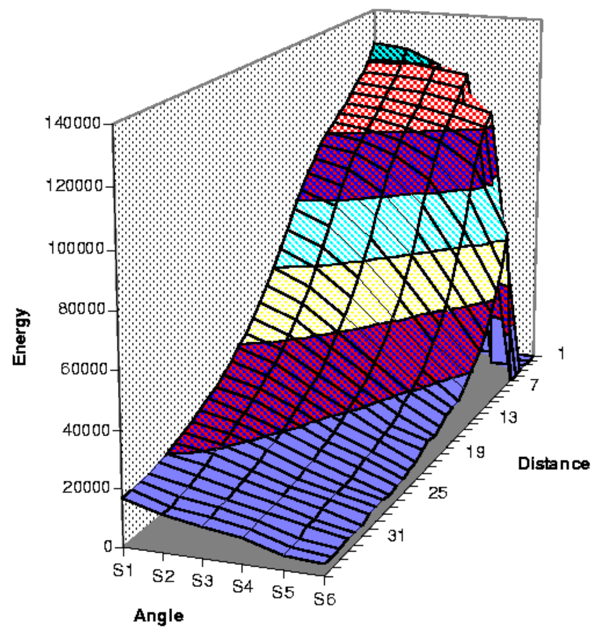
The response to white copy paper
(a dull, reflective surface)

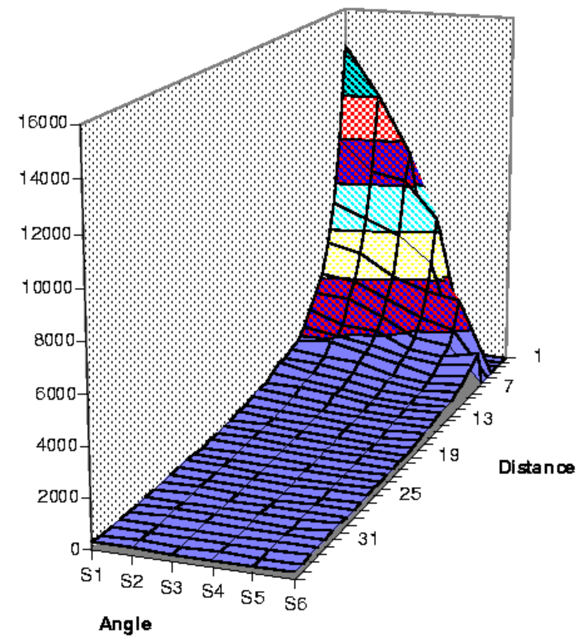
raw values
(put into 4 bits)



inches

15º increments



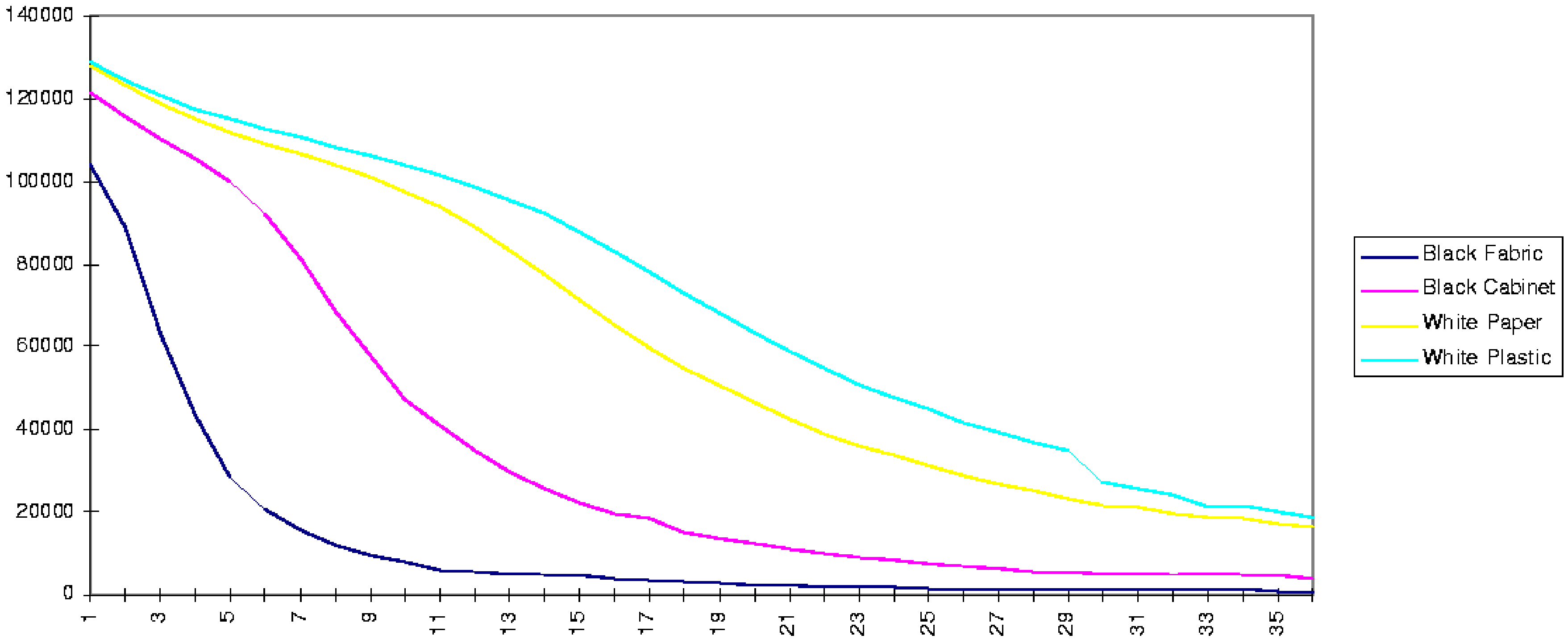


in the dark

fluorescent light

incandescent light

Infrared calibration



energy vs. distance for various materials

(the incident angle is 0º, or head-on)

(with no ambient light)

Sonar sensing

single-transducer sonar timeline

0

a “chirp” is emitted into the environment

75μs

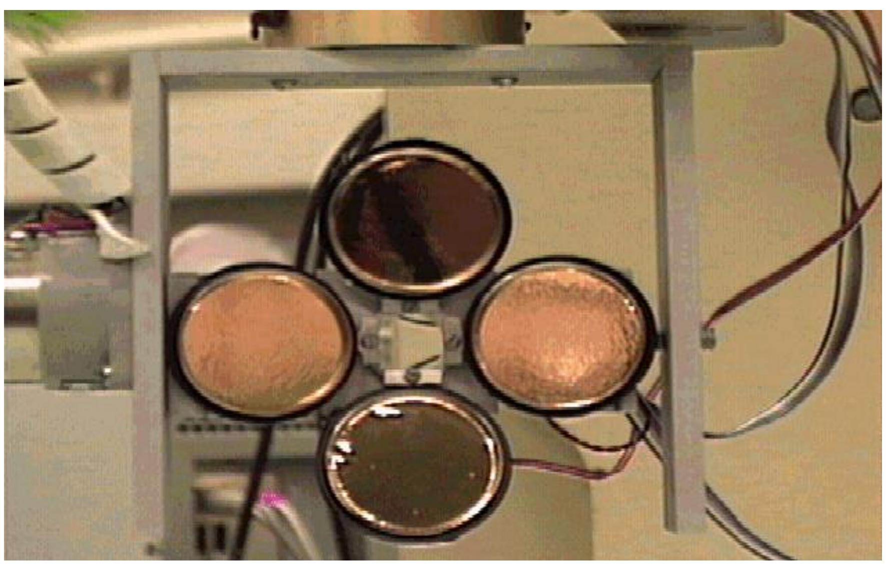
typically when reverberations from the initial chirp have stopped

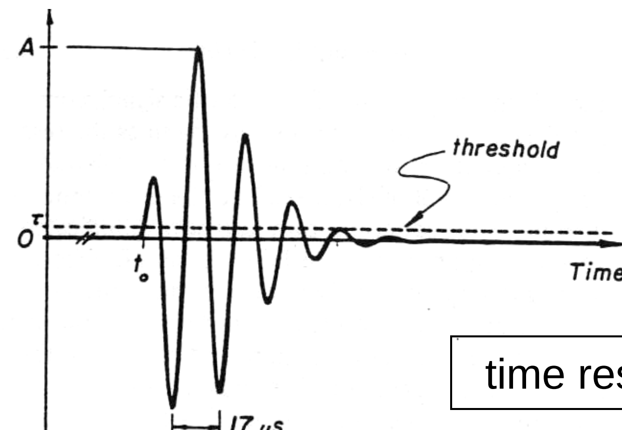
the transducer goes into “receiving” mode and awaits a signal...

limiting range sensing

.5s

after a short time, the signal will be too weak to be detected





time response

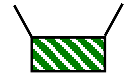
blanking time

Polaroid sonar emitter/receivers

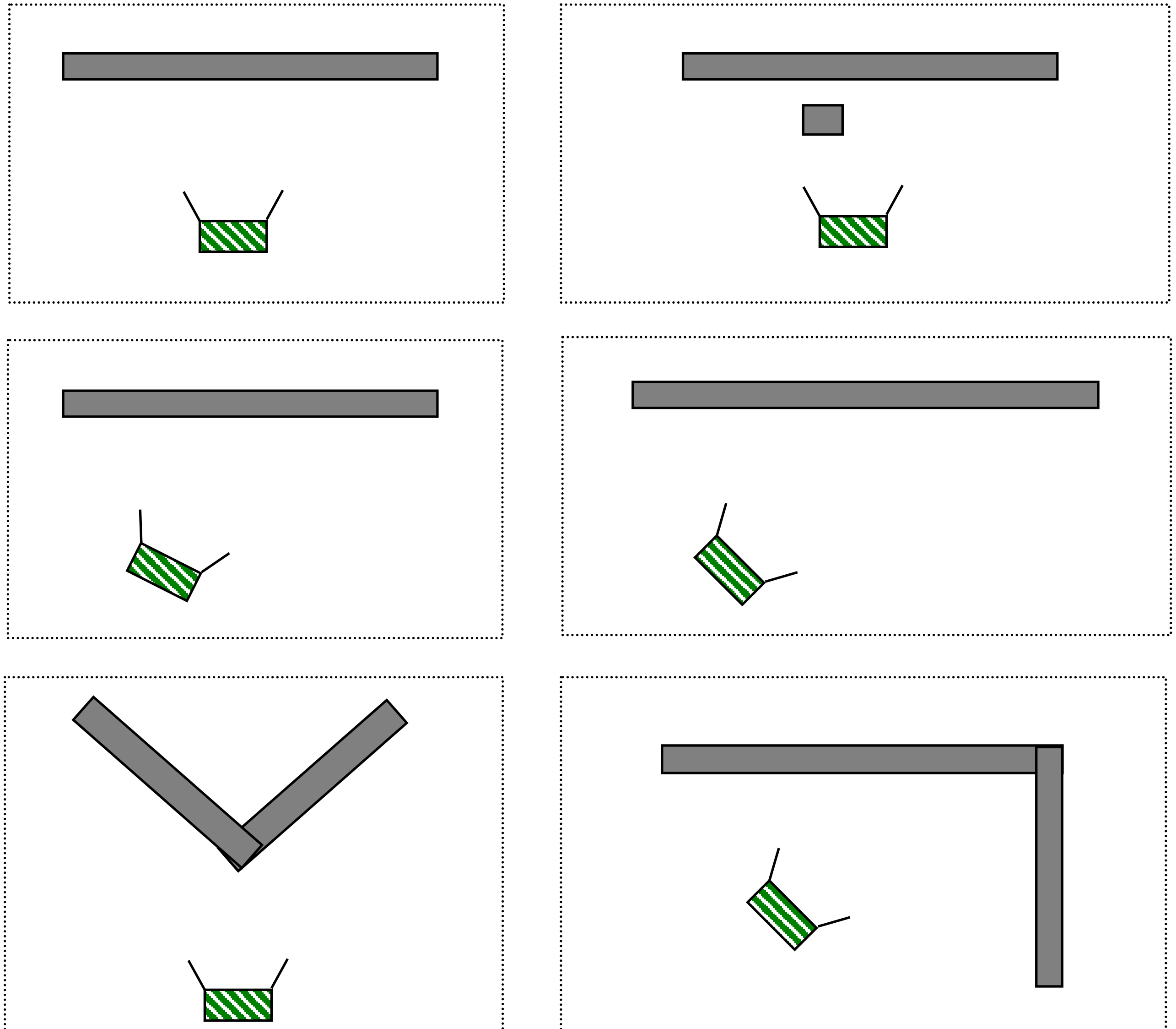
No lower range limit for *paired* sonars...

walls
(obstacles)

Sonar effects



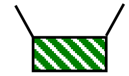
sonar



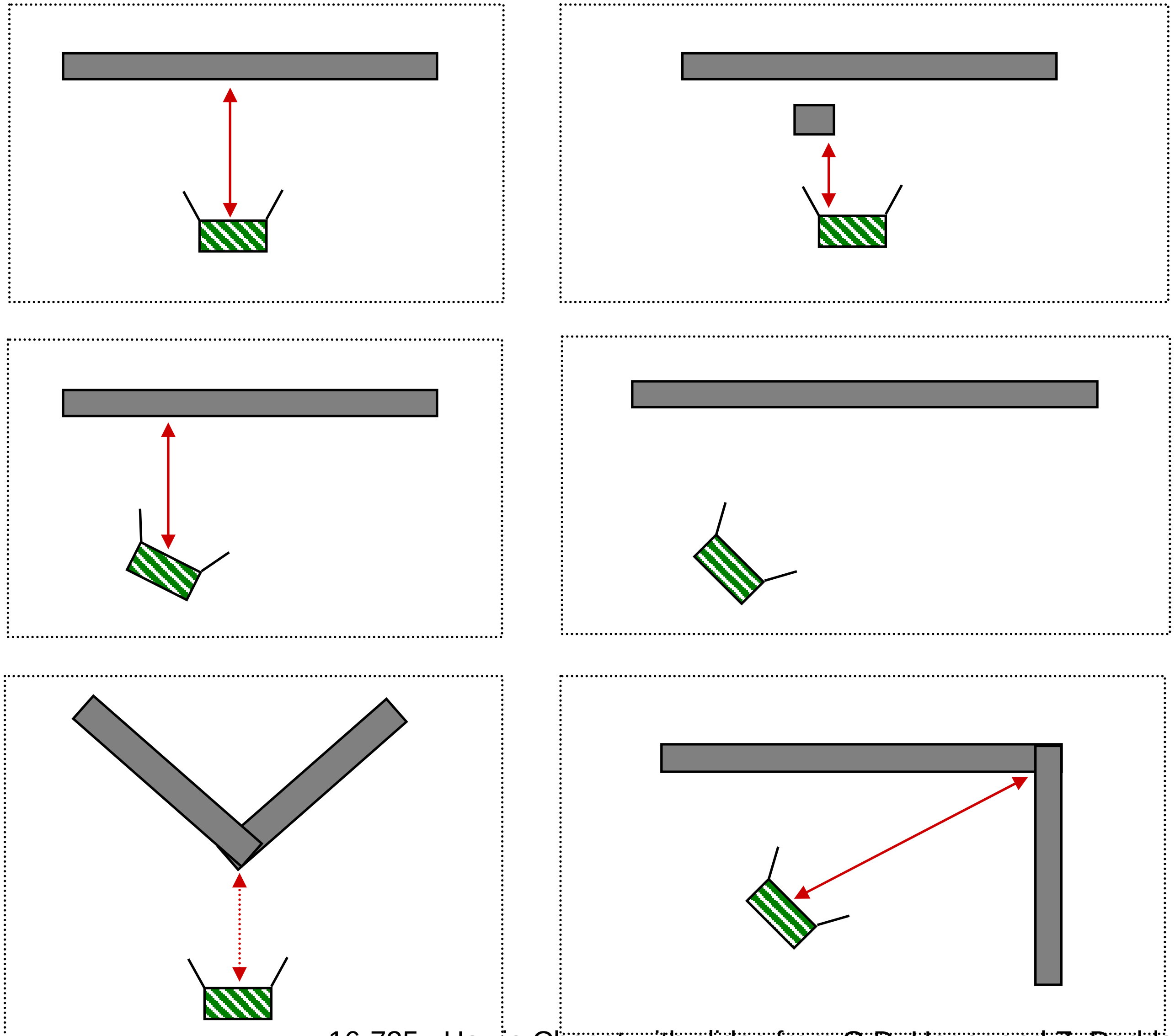
Draw the range reading that the sonar will return in each case…

walls
(obstacles)

Sonar effects



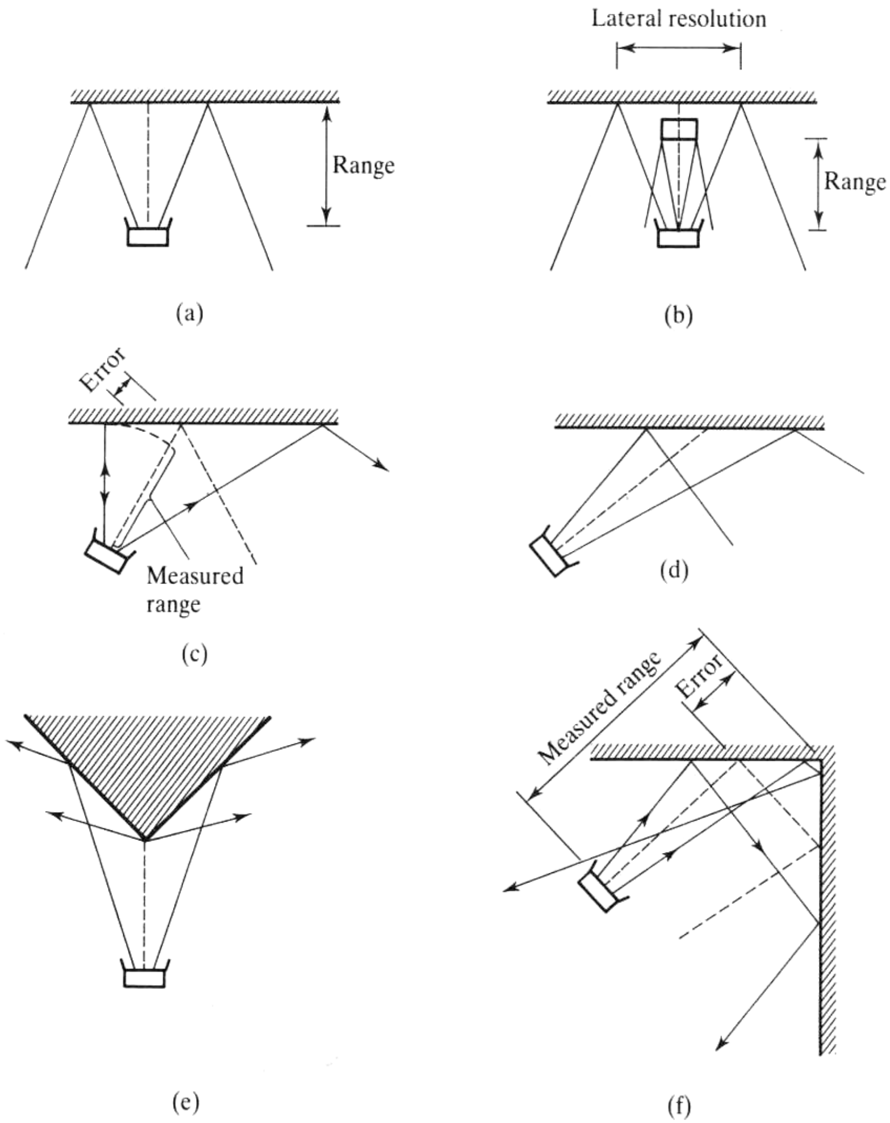
sonar



Draw the range reading that the sonar will return in each case…

holding a sponge…

Sonar effects



(a) Sonar providing an accurate range measurement

(b-c) Lateral resolution is not very precise; the closest object in the beam's cone provides the response

(d) Specular reflections cause walls to disappear

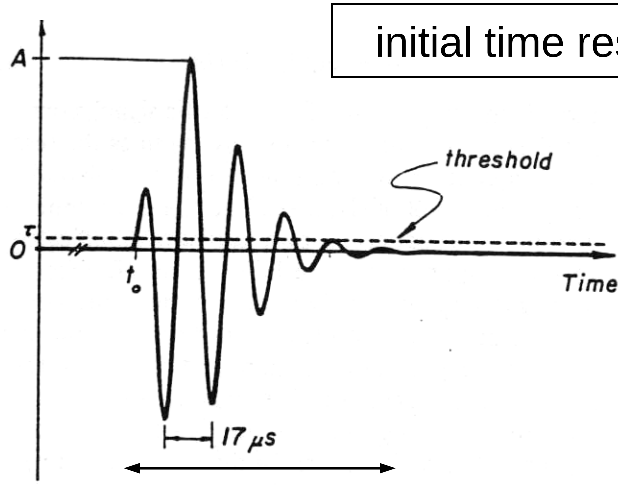
(e) Open corners produce a weak spherical wavefront

(f) Closed corners measure to the corner itself because of multiple reflections --> sonar ray tracing

resolution: time / space

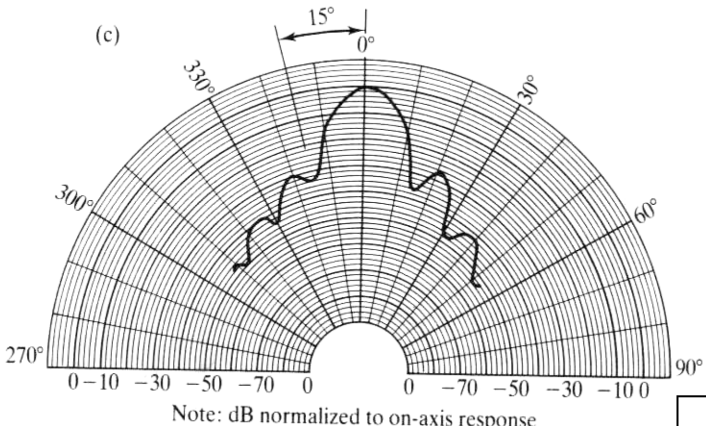
Sonar modeling

initial time response

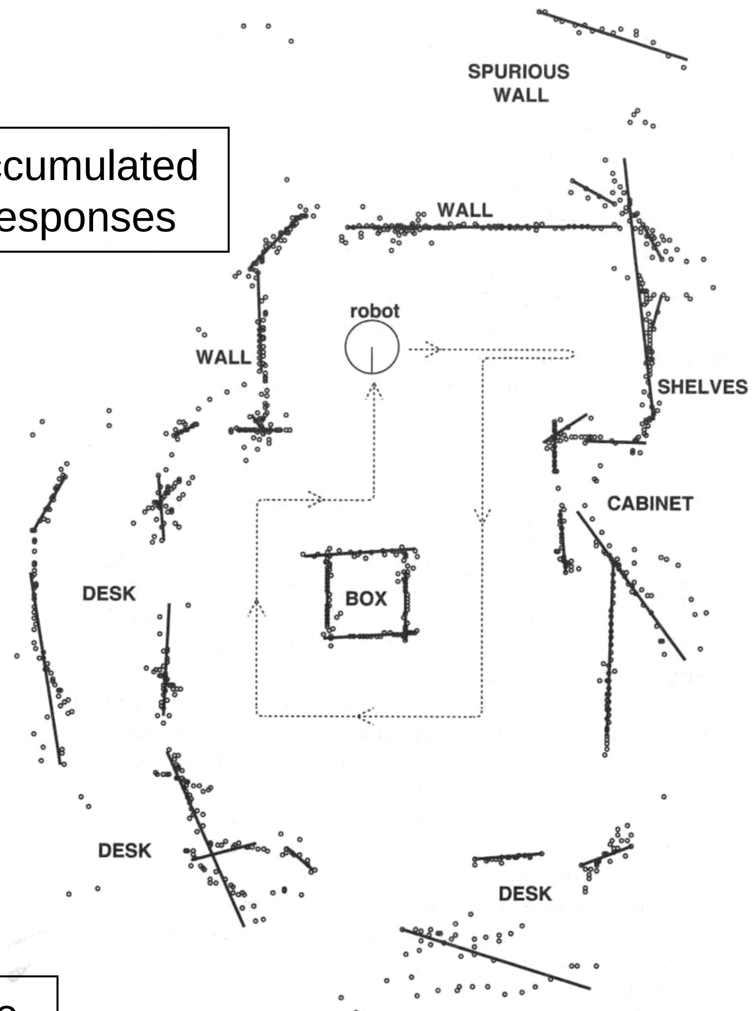


blanking time

cone width



accumulated responses

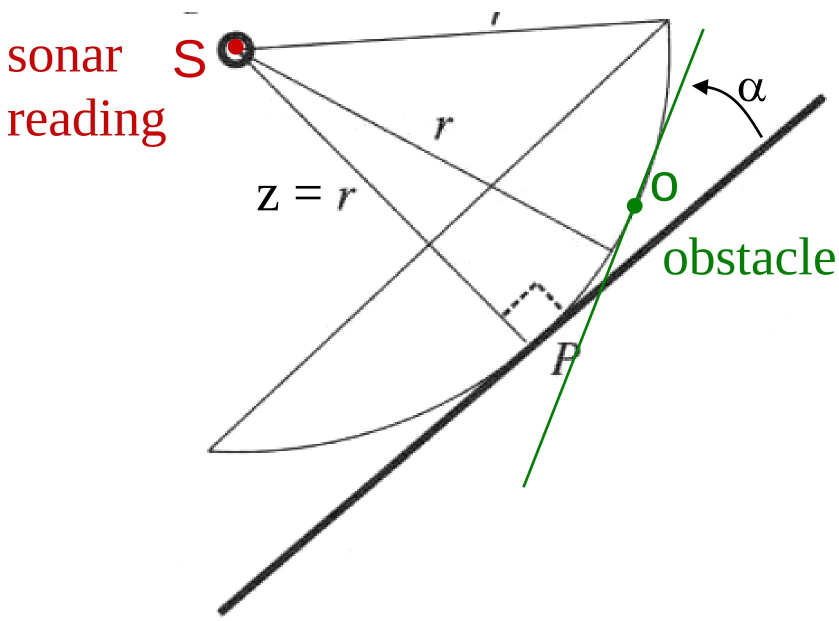


spatial response

Sonar Modeling

response model (Kuc)

$$h_R(t, z, a, \alpha) = \frac{2c\cos\alpha}{\pi a \sin\alpha}\sqrt{1 - \frac{c^2(t - 2z/c)^2}{a^2 \sin^2\alpha}}$$



- Models the response, h_R, with

  c = speed of sound

  a = diameter of sonar element

  t = time

  z = orthogonal distance

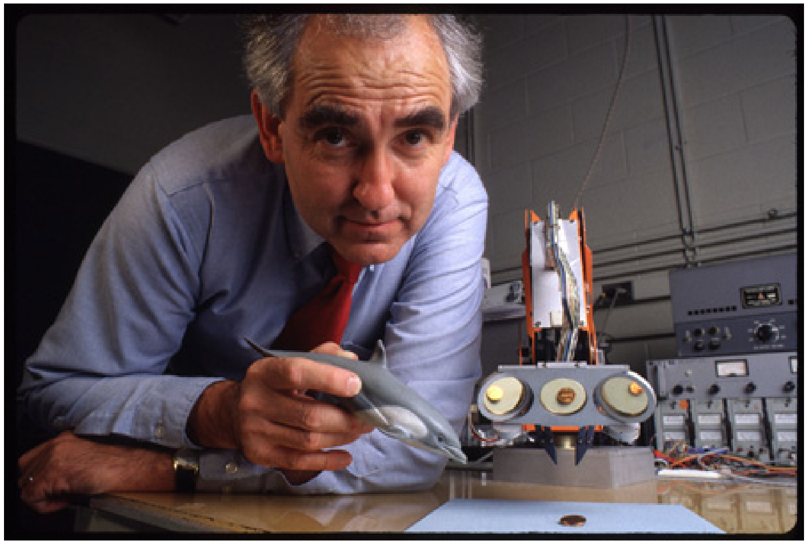
  α = angle of environment surface

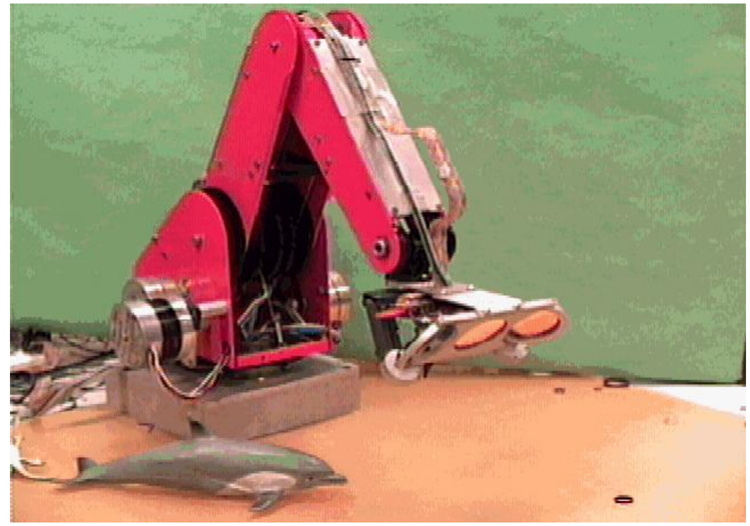
- Then, allow uncertainty in the model to obtain a probability:

$$\boxed{p(S \mid o)}$$

chance that the sonar reading is S, given an obstacle at location o

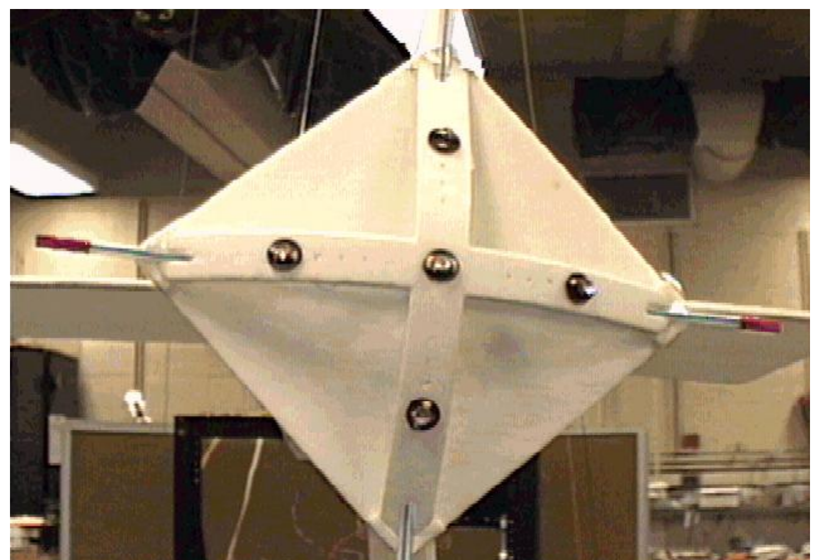
Roman Kuc's animals





Rodolph

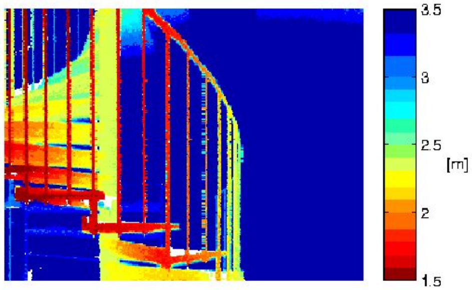
Robat



Laser Ranging

LIDAR

Sick Laser



LIDAR map

Sick laser
range finder

Summary

- Bug 1: safe and reliable
- Bug 2: better in some cases; worse in others
- Should understand the basic completeness proof
- Tangent Bug: supports range sensing
- Sensors and control
  - should understand basic concepts and know what different sensors are