Joseph J. Viscomi and Henry Hexmoor

Towards Quantification of Externality in Collaborating Communities

Abstract. This paper presents an analysis of a multi-agent system intended to model total task completion rates in collaborating communities. The spillover effect is emergent. We begin to quantify externality of marginal utilities. We are concerned with maximizing the output of the given community through regulation of agent to task ratios. We outline emergent externalities when collaboration occurs, even in absence of communication and planning. We offer a simple means of regulating externalities in these situations by means of sketching an agent distribution strategy.

Introduction

It is in the best interest for governing bodies and organizations to seek to maximize their community output. Traditionally, theoretical models used to analyze complex social systems come from social sciences using qualitative as well as empirical methods. Novel quantitative models are emerging; in particular, models based in the theories of complexity and emergent phenomena (Castillo, 2009). One mechanism that increases output is to explore synergistic collaboration. Synergies are often evaluated with economic analysis to discover marginal utilities.

Reasoning about marginal utilities provides a predictive power about work force management. Equilibrium in a perfectly competitive economy is a situation of Paretian optimum, except when there is interdependence among the members of the economy that is direct, in the sense that it does not operate through the market mechanism (Scitovsky 1954). In studying collaboration and optimality there exists interdependence among agents. The output of an agent is not only depended by his own input when collaboration exists, but also depends on the input of all the agents which are collaborating on the same goal or task. In general equilibrium theory, then, direct interdependence is the villain of the piece and the cause for conflict between private profit and social benefit (Scitovsky 1954). This can be generalized out to a concept known in economics called externalities and must be introduced here to adequately determine group and community efficiency and at what cost. It will become apparent that when collaboration exists externality is a very important measure and cannot be ignored when determining optimality with hidden information among agents in a non-deterministic environment. Collective opinions have been explored from a control theory perspective (Stefanuk, 2010). Our recent work has shown that as the collaboration among agents increases, the performance of the agent community increases linearly and the optimal performance of the agent community is achieved when the collaboration among agents is 100% (Hexmoor, 2001). Furthermore, when the number of tasks assigned is far fewer than the number of agents in the community, the optimal performance of the agent community is achieved at nearly 100% collaboration rate. We have developed a series of multiagent simulations for a domain neutral environment where agents performed abstract tasks. The task selection process has three variants based on how the agents collaborate; *min-collaboration*, *random-collaboration*, *max-collaboration*. The *min-collaboration* and *max-collaboration* are greedy algorithms because the try to maximize or minimize distribution of agents among tasks respectively.

Using this platform we replicated marginal gains that support the phenomenon of externality described in the next section.

1. Observations

First we explored marginal utility i.e., the benefit, of continually adding an agent to a given task, (shown in Figure 1). The number of tasks is set to one and the number of agents start at one and will be incremented by one in each step until the average rate of task completion reaches 100%, the community-collaboration-percentage is set to 100%. Obvious observation is that the added utility rapidly diminished and the additions did not have a linear effect on the output. Instead, it approximates a logarithmic function. This is modeled as a function that gives the probability that the given task will be completed, noted: $P_\tau(\lambda)$. It is apparent that this is a monotonically increasing function, i.e., $P_\tau(\lambda) \Leftarrow P_\tau(\lambda+1)$ holds.

Computing the average probability of any given task to be completed is then simply given by (equation 1):

$$\frac{\sum_{\tau=1}^{n} P\tau(\lambda_a)}{n}$$

Where n is the number of tasks and λ_a is the number of agents out of the community collaborating on the given task τ.

It is clear that the most positive marginal utility gain is realized when two agents are collaborating. Figure 2 shows the marginal utility with increasing group size.

Figures 1 and 2 support the fact that as a larger number of agents collaborate the upper most agents, i.e. 16 and higher would clearly benefit the overall

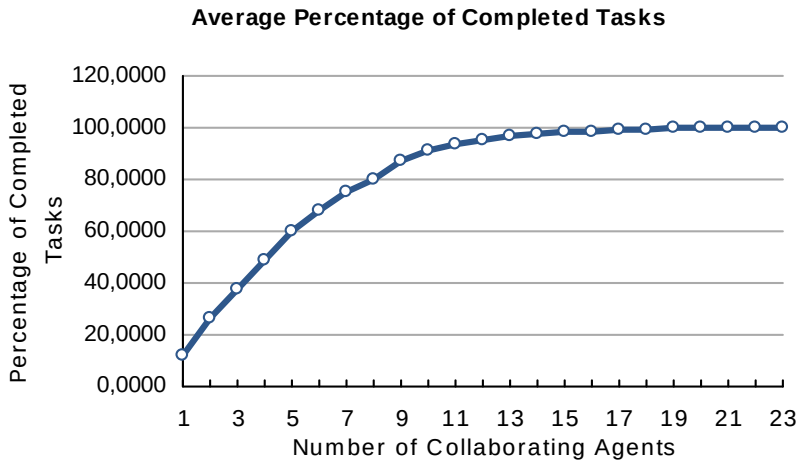

Figure 1. Collaboration versus Task completion

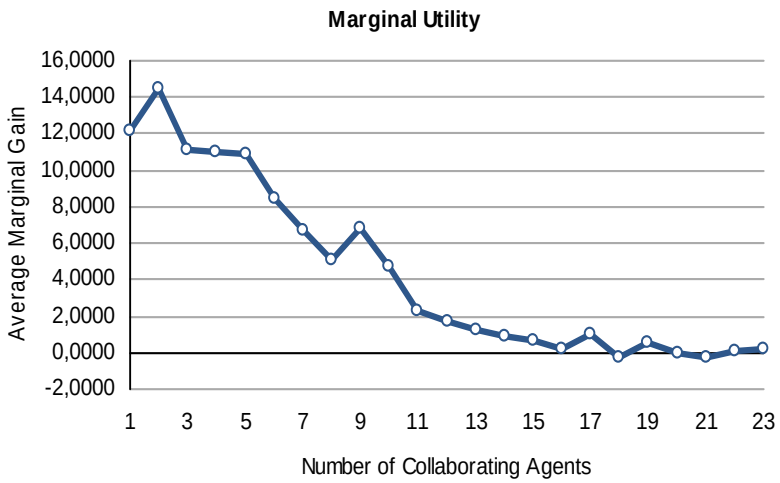

Figure 2. Computing the difference between the points is shown in Figure 1

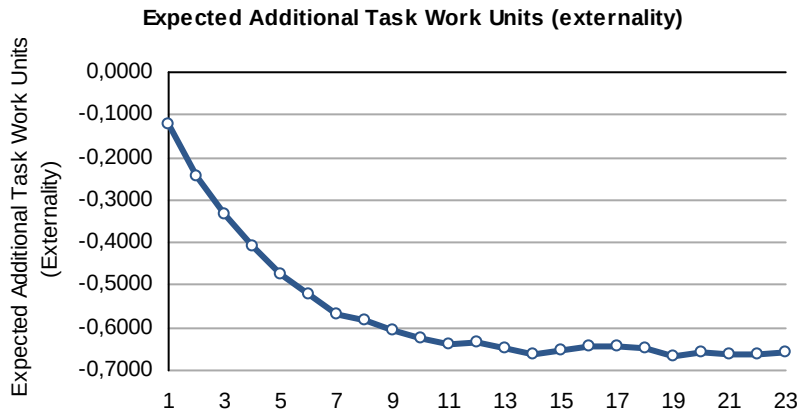

Figure 3. Negative Externality

production of the community by working on other tasks with lower collaboration levels. This corroborates Gossen's First Law of diminishing marginal utility-- marginal utilities diminish across the range relevant to decision making (Gossen, 1984). This brings us back to the concept of externality as discussed earlier and illustrated in Figure 3.

We captured negative externality and show that as this number gets more negative agents are working less efficiently. The correct way to interpret these numbers is they indicate a minimum gap from the work input into the system to the actual work completed in the system. Figure 3 strongly shows that as you add more agents the rate of added benefit to the community is decreasing; hence, the negative externality. It would also seem to suggest that the negative externality has a lower limit, but we believe it is because not all the agents are actively working since the task is completed before other agents have an chance to contribute. Thus far the data supports a saturation point when collaborating is occurring, but not enough information exists yet to draw a complete conclusion. Despite $P_{\tau}(\lambda)$, depending on the number of tasks in the community and how the agents are assigned to tasks can lead to guaranteed failures in the system. When there are 1000 tasks and 100 agents, the plot of guaranteed failures for all three mechanisms of task selection are shown in Figure 4.

The function that generates the probable number of guaranteed failures is:

$$P_{\omega}(A,T)\begin{cases} A = \{A : A \in N, A > 0\} \\ T = \{T : T \in N, T > 0\} \end{cases}$$

A is the number of agents in the community, and T is the number of tasks needed to be completed. We note that when the task selection process attempts to maximize collaboration, the number of guaranteed failures accelerates and is not a linear correlation. This is intuitive enough because as you group more agents together there are less agents available to work on more tasks, therefore, you will have more tasks not being worked on and thus increase the number of guaranteed failures which is illustrated in Figure 4.

Equation 1 simplifies this system too much, by evenly distributing the probabilities, and it should be evident from Figure 4 that this is not the case. There is a contradiction that exists then because tasks which have a guaranteed failure have no probability of getting completed, and with this simple reasoning we can modify equation 1 to the following (equation 2):

$$\frac{\sum_{\tau=1}^{n} P\tau(\lambda_a)}{n - P\omega(A,T)}$$

Equation 2 takes into account both probability of task completion and probability of task guaranteed failures and distributes it accordingly.

The dramatic effect of producing guaranteed failures by means of maximizing the collaboration levels is shown in Figure 5 by the output of the community.

Conclusions

This paper has outlined salient governing attributes in a multi-agent system to optimize group formation. These results are consistent with the Ronald Coase's seminal work in his 1960 paper titled "The

Problem of Social Cost". Simply stated, there exists a saturation point when the marginal utility of collaboration approaches zero. It is apparent that given the marginal utility that agents on the upper ends of collaboration levels would achieve a better performance for the community as a whole if they were redistributed to other tasks. This illustrates that the major controlling attribute is ultimately the distribution of agents among the tasks or in this case the task selection algorithm. The safe task selection algorithm out of the three that were tried was the random-collaboration-task-selection. Meaning that if you have no means of regulating collaboration random always produces more output than no cooperation. However, if you have mechanisms for regulating groups, even as simply as regulating the percentage out of the community involved. Then it is best to maximize the group to around 70% collaboration levels to maximize the output of the community (Figure 5). It is also apparent that by regulating how the tasks are selected you can control the externalities in the system, if this is a concern to you or if you need to evaluate the benefit to the system as a whole. You also cannot just simply look at output when determining efficiency of a community when collaboration occurs, because as shown much waste and spillover can occur when there is interdependence among agents. Externality is needed to de-

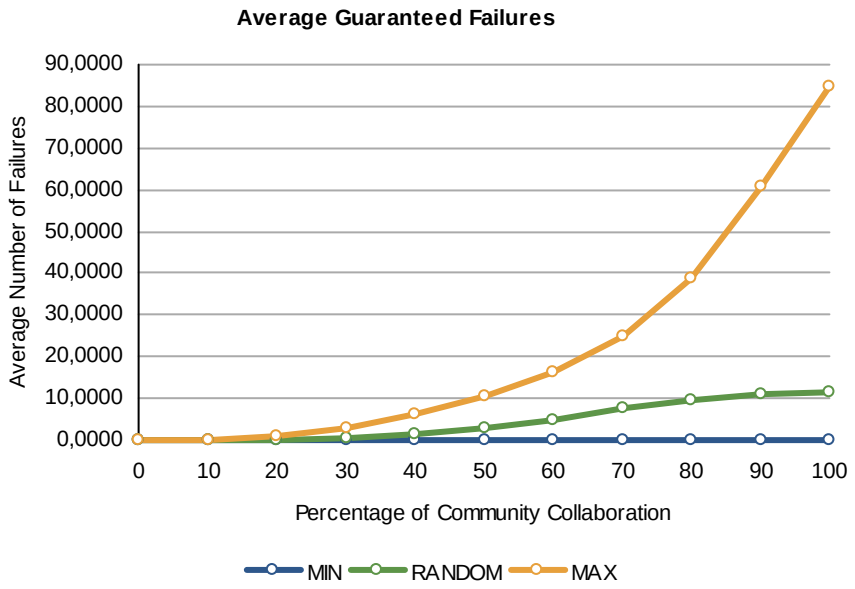

Figure 4. Failure Rates

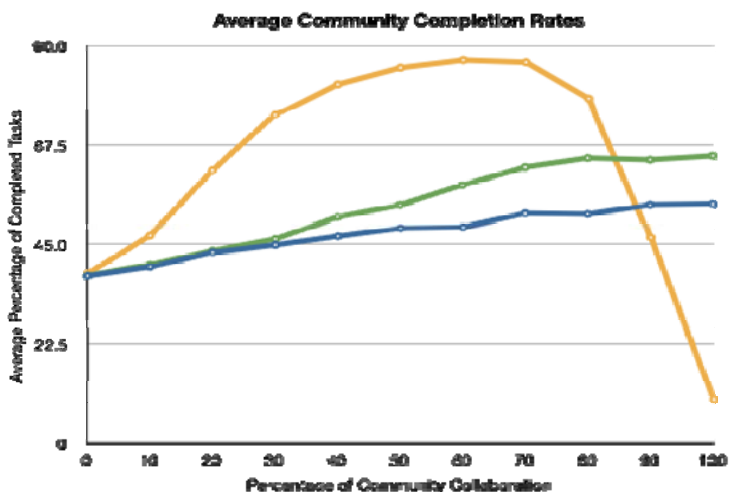

Figure 5. Community Output Rates for all three agent distribution strategies

scribe how efficient the agents are working relative to the social gain of the community. Further examination is required for coming up with an optimal task selection algorithm that maximizes the output to the absolute maximum. This simulation is also based on the simple fact that all agents posses a probability when being created that they will be able to complete any given task individually, or in this model collaboration is not required for agents to complete a task in one unit of time. It would be interesting to know what the implications would be when the tasks become more complex and require collaboration, what is the relationship between group size and task complexity. Our larger stride is to devise methods for dynamically observe and predict externality so it can become a parametric tool.

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Joseph J. Viscomi and Henry Hexmoor. Department of Computer Science, Southern Illinois University, Carbondale, IL 62901 {jviscomi, hexmoor}@cs.siu.edu.